1 Types of Equality

1.1 Mathematical Equalities

• = Identity

Two symbols or names represent the same mathematical “object” or “concept”:

\[ a \text{ is my height,} \]
\[ b \text{ is the distance from the bottom of my feet to the top of my head when I am standing} \]
\[ a = b \]
\[ 0.5 = \frac{1}{2} \]
\[ \sqrt{2} = 1.41421... \]

To realize and believe that these two expressions represent the same thing requires two very different sets of knowledge particular for each expression. (Decimal expressions and expansions, and, perhaps, quadrature. For fractions, the equivalence classes of fractions.)

(Does this always mean that these two symbols merely share the same definition? Or could \( \int_0^1 x \, dx = 0.5 \) be an example?)

• ≡ (or sometimes := and perhaps =:) Definition

Definition of a particular notation or symbol:

\[ \sum_{R^n_1}^s e^{-\beta \left( n_1^{R^n_1} \epsilon_1 + n_2^{R^n_2} \epsilon_2 + \ldots \right)} \equiv \sum_{R^n_2} \exp \left[ -\beta \sum_{r \neq s} n_r^{R^n_r} \epsilon_r \right] \]

One could define a “direction” using the colon:

(New symbol/notation) := (Expression using terms that are usually already defined)

(Expression using terms that are usually already defined) =: (New symbol/notation)

• = Quantitative Equality

The values (or measures) of two different mathematical objects or quantities are (proposed or theorized to be) the same:

\[ E = \gamma mc^2 \]

• = Significance Equality

(Equality of Significance, or Inequality of Insignificance)

Two numbers are equal up to the last significant figure:

\[ 1.2345678 = 1.2345789 \]
• \(\approx\) (or \(\cong\)) Approximate Quantitative Equality (or Closeness in some other sense)

In science this is what the equals sign really means – equality within some margin of error.

(I sometimes use \(\cong\) to mean a closer approximation that \(\approx\).)

• \(\doteq\) Representation (an “abbreviated equality”)

A symbol or object represents another mathematical object. (The representative may play the same algebraic role as the represented object in some other space of objects. Or there may be some more complicated relationship or equality that links the objects.)

\[
|\psi\rangle \doteq \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}
\]

while

\[
|\psi\rangle = \begin{pmatrix} |\psi_a\rangle \\ |\psi_b\rangle \\ |\psi_c\rangle \\ |\psi_d\rangle \end{pmatrix}
\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = a|\psi_a\rangle + b|\psi_b\rangle + c|\psi_c\rangle + d|\psi_d\rangle
\]

• \(\sim\) Equivalence (Classes)

Two objects are members of the same equivalence class:

\[\frac{1}{2} \sim \frac{2}{4}\ or\ \frac{1}{2} = \frac{2}{4}\]

• \(\equiv\) Indication of the domain of a function

\(f = f(x, y)\) lets you know that \(f\) is a function of two variables, \(x\) and \(y\)

• Big O notation

\(T(n) = O(n^2)\) makes sense but \(O(n^2) = T(n)\) is meaningless

• Various “equals” signs:

\[=, :=, =:, \doteq, \doteqdot, \equiv, \backsim, \backsimeq, \bumpeq, \Bumpeq, \circeq, \curlyeqprec, \curlyeqsucc, \doteqdot, \eqcirc, \fallingdotseq, \precapprox, \preccurlyeq, \precsim, \risingdotseq, \shortparallel, \smallfrown, \smallsmile, \succapprox, \succcurlyeq, \succsim, \thickapprox, \thicksim\]

I’d use the following commands, but they require the mathabx package, and that package tries to define things that already exist.

\[\coloneq, \topdoteq, \botdoteq, \dotseq, \eqcirc, \fallingdoteq, \risingdoteq, \corresponds\] (plus some that I accidentally deleted.)
1.2 Programming “Equalities”

- = Assignment
  \( x = 5 \) sets the value of \( x \) as 5.

- == Test for Equality
  Boolean logic binary relation that tests for equality. If \( a = b \), then \( a == b \) returns TRUE (1), and if \( a \neq b \), then \( a == b \) returns FALSE (0).

2 Types of Solutions

- solution
  plug it in, see that it works

- formal-solution
  also has to be well-defined (i.e., convergent not divergent, ...?)

- well-defined solution
  (convergent...?)

3 Integration Notation

- Writing the differential immediately after the integral sign makes the analogy with the summation notation more apparent:
  \[
  \sum_{k=0}^{n} f(k) \rightarrow \int_{0}^{n} dk \ f(k)
  \]

- Writing the differential at the end of the integrand can provide clarity in delineating where the integrand ends and another term begins:
  \[
  5 \left( \int d(x(f(x + y) - 5 + h(x, z)) + g(x)) \right) - k(x, z)
  \]
  \[
  5 \left( \int (f(x + y) - 5 + h(x, z)) \ dx + g(x) \right) - k(x, z)
  \]

- One may make mistakes using the first notation with multiple integration: separating the Jacobian into pieces may take some attention
  \[
  \int_{P} f(\mathbf{r}) \ d\mathbf{a} = \int_{S} f(\mathbf{r}) \ r \ dr \ d\phi = \sum_{\text{simplices}} \int \ r \ dr \int \ d\phi \ f(\mathbf{r})
  \]
  \[
  \int_{V} f(\mathbf{r}) \ dv = \int_{V} f(\mathbf{r}) \ r^2 \ sin \theta \ dr \ d\theta \ d\phi = \sum_{\text{simplices}} \int r^2 \ dr \int \ sin \ \theta \ d\theta \int \ d\phi \ f(\mathbf{r})
  \]

- There may be times where any of these notations are most appropriate:
  \[
  \int_{P} \text{ or } \int_{\Gamma} \text{ or } \int_{C} \quad \int_{P} \mathbf{F} \cdot \mathbf{ds} = \int_{P} \mathbf{F} \cdot \mathbf{dl} = \int_{P} \mathbf{F} \cdot \mathbf{dx}
  \]
− $P$ stands for path, $\Gamma$ is a conventional name for a path, and $C$ stands for contour, which implies that it is a boundary of some figure, body, or surface ($C = \partial S$)
− $s$ could be confused with the cylindrical $s = s\hat{s}$ and
− $x$ could be confused with the Cartesian $x = x\hat{x}$

$$\int_S d\alpha \cdot F = \int_S d\alpha \hat{\alpha} \cdot F = \int_S d^2 x \hat{\alpha} \cdot F$$
$$\int_V dv \ F = \int_V d^3 x \ F$$

4 Characters and Variables

This may be going a bit over-board, but I propose some new characters and conventions:

- “scripty rho” for energy density: (ooh, notice a hint of an “e” in the character)

To be distinguished from rho for charge density:

- “scripty J” for energy current: (rather than $\vec{J}$)

To be distinguished from $J$ for charge current density: