1. *Electricity and Magnetism* (Spring 1996)

A waveguide has dimensions (10 cm) $\times$ (5 cm) in the xy-plane and is infinitely long in the z-direction.

(a) What is the lowest frequency, f_1, in GHz (or 10^9 Hz), at which the waveguide will permit propagation of radiation?

(b) At a frequency of $5f_1/4$ what is the behavior of the electric field as a function of x, y, z, t?

(c) What are the group and phase velocities?

2. *Electricity and Magnetism* (Spring 1996)

Consider a circular line charge of radius a in the xy-plane having a charge density:

$$\lambda(\phi) = +\lambda; \qquad 0 < \phi < \pi$$

$$\lambda(\phi) = -\lambda; \qquad 0 < \phi < 2\pi$$

where $\phi = \arctan(y/x)$. Calculate the monopole moment, the dipole moment and all components of the qudrupole moment tensor (i.e., Cartesian multipoles Q_{ij}) for this charge distribution and thus demonstrate that the far-field potential for this charge is given by:

$$\phi(x, y, z) = 4\lambda a^2 \frac{1}{r^2}\left(\frac{y}{r}\right) + \mathcal{O}\left(\frac{1}{r^4}\right), \qquad \text{for } r = \sqrt{x^2 + y^2 + z^2} \gg a$$

3. *Statistical Mechanics and Thermodynamics* (Spring 1996)

A system with the ideal gas equation of state is contained in a rectangular box of length L (along the x axis) and cross-sectional area A. Initially the temperature is T_1, a constant, and pressure varies linearly from p_1 at one end to p_2 at the other ened of the box. The molecular weight of the gas is M. What are the energy E, temperature T, and pressure p in equilibrium? The box is an isolated-closed, insulated system.

4. *Statistical Mechanics and Thermodynamics* (Spring 1996)

Consider a long stretched string with tension τ and mass per unit length μ at thermal equilibrium at temperature T. What is the r.m.s. displacement of the string due to thermal noise between frequencies f_1 and f_2?

5. *Electricity and Magnetism* (Spring 1996)

Consider an infinite cylindrical column of negatively charged particles moving with a constant velocity $\mathbf{v} = v\hat{\mathbf{z}}$ and having volume charge density, $\rho(r) = \rho_0\, e^{-r/a}$, where $r^2 = x^2 + y^2$. (A stationary uniform background of neutralizing positive charges is also present.)

(a) Calculate the current crossing an xy-plane.

(b) Calculate $\mathbf{B}$ everywhere.

6. *Electricity and Magnetism* (Spring 1996)

Consider a point charge Q_0 a distance $2a$ from the center of a conducting isolated sphere of radius a.

Hint: Use image method.

(a) Calculate the net charge on the sphere and the electrostatic potential of the sphere ($V(\infty) = 0$) if the force on Q_0 is zero.

(b) Calculate the force on Q_0 if the net charge on the sphere is zero.

7. *Electricity and Magnetism* (Spring 1996)

A soft photon of energy (1 eV) collides head-on with an electron of (total) energy (10 GeV). What is the energy of the photon after scattering by 180°? Assume $m(e^-) = (0.5\ \mathrm{MeV/c^2})$.

8. *Quantum Mechanics* (Spring 1996)

Consider a particle of mass m in the one-dimensional square well potential:

$$V(x) = \begin{cases} -V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

(a) Find the form of the wavefunction inside and outside the well in terms of the positive constants $\beta = \sqrt{2m|E|}/\hbar$, $\kappa = \sqrt{2m(V_0 - |E|)}/\hbar$, where $E = -|E|$ is the energy of the bound state.

(b) Show that there is *always* at least one bound state; and find the condition for a second bound state.

9. *Statistical Mechanics and Thermodynamics* (Spring 1996)

Consider 2 identical Bosons of mass m in a one-dimensional box of length L at temperature T. Find the equilibrium quantum statistical mechanical probability that a measurement finds both particles in the left half of the box. You can express your answer in the form

$$\sum_{j,k} f_{jk}$$

where f depends upon T, L, m, and universal constants.

10. *Quantum Mechanics* (Spring 1996)

The state function of an electron is given by

$$\psi = R(r)\left[\sqrt{\frac{1}{3}}\,Y_1^0(\theta,\phi)\,\chi_+ + \sqrt{\frac{2}{3}}\,Y_1^1(\theta,\phi)\,\chi_-\right]$$

with

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi},$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(a) What is the total angular momentum and its z-component?

(b) What is the probability density for finding the electron with spin up at r, θ, ϕ? With spin down?

(c) Show that the probability for finding the electron at r, θ, ϕ, summed over its spin, is spherically symmetric (that is, independent of θ and ϕ).

11. *Statistical Mechanics and Thermodynamics* (Spring 1996)

A black spherical satellite of radius r is at a distance D from the sun ($D \gg r$). Assume that the sun radiates as a blackbody of temperature T_0 and subtends an angle of α radians as seen from the satellite. What is the steady state temperature of the satellite?

12. *Quantum Mechanics* (Spring 1996)

Consider an atom which can be one of two states: the ground state $|0\rangle$ and the excited state $|1\rangle$. The lifetime of the excited state is τ. (This is called a *Poisson process.*) A laser acts on the atom so as to immediately return it to the excited state after each decay. What is the probability distribution for the occurrence of two decays within time t?

13. *Quantum Mechanics* (Spring 1996)

Derive the dipole-dipole magnetic interaction energy for eigenstates of the *total* spin of two protons. The Hamiltonian is of the form

$$H = A(\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2) + B(\boldsymbol{\mu}_1 \cdot \mathbf{r})(\boldsymbol{\mu}_2 \cdot \mathbf{r})$$

where A and B are two constants, $\mathbf{r}$ is the distance vector separating the two protons, and $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are the magnetic dipole moments of the two protons. The proton magnetic moment is related to its spin via $\boldsymbol{\mu} = \mu_p \mathbf{S}$.

14. *Quantum Mechanics* (Spring 1996)

Consider the three-dimensional harmonic oscillator described by the Hamiltonian

$$H_0 = \frac{1}{2m}({p_x}^2 + {p_y}^2 + {p_z}^2) + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2).$$

A small perturbation $V = \lambda xyz$ is introduced. Compute to lowest non-vanishing order the shift in energy of the ground state of the unperturbed Hamiltonian.

Hint: For a one-dimensional harmonic oscillator $x = (a + a^\dagger)/\sqrt{2m\omega}$, where a and $a^\dagger$ are lowering and raising operators.