Review of Winter 2008 Lecture Series

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⋆ ⇒ original material produced/proposed

- ⋆ Week 1: Explicitly worked out what is meant by "a symmetric, traceless (2×2) tensor has a period of 180°."
- • Week 2: Found how symmetric, traceless 2×2 tensors relate mathematically to spin-2 radiation deformations (as in the deformation of matter due to gravitational waves). Chris found a very important paper, "What is Spin?" by Hans Ohanian, giving a rare, physical explanation of spin.
- • Week 3: Found how symmetric, traceless 2×2 tensors relate via Einstein's field equations to spin-2 radiation deformations.
- ⋆ Week 4: Posed the fundamental questions whose answers are my first goal in understanding QFT. Proposed an answer to "What information does a QFT state contain?" and various notations to symbolize that information, including this "Fock ket":

$$\| \, \Psi(t) \, \rangle\!\rangle = \Big(\big|\Psi_1(t)\big\rangle \, , \big|\Psi_2(t)\big\rangle \, , \ldots , \big|\Psi_n(t)\big\rangle \, , \ldots \Big)$$

  In courses on QFT, very little, if any, mention is made of actual states and how one might graph or plot them. That is one of our goals.
- • Week 5: Showed how one discovers particle creation and annihilation operators for the Schrödinger and Klein-Gordon equations of motion. I also showed how Fourier transformation relates various objects, such as $|\mathbf{x}\rangle$ and $|\mathbf{p}\rangle$.
- ⋆ Week 6: Found a way to express the (eigen) wave-functional $\Psi_1[\varphi]$ (from Hatfield Eqn. 10.30) more generally in terms of wave-functions and Dirac bras and kets.
- ⋆ Week 7: Derived the Klein-Gordon equation from a classical physical model. This lead to an explanation that I've never seen in a textbook: the dispersive nature of the Klein-Gordon equation allows signals with frequency/energy greater that mc^2 to propagate freely but causes signals with lower energy to decay. I showed that the decaying signals do not seem to correlate to virtual particles.
- • Week 8: Derived the Klein-Gordon Green's function / propagator. This was in an attempt to answer a question that turned out to be too advanced for me right now: how does the Klein-Gordon Green's function / propagator relate to raising/lowering of the field (in a classical sense, in particular)? Also, I tried (mostly unsuccessfully) to find out what the general solutions to the Klein-Gordon equation are, for reference when studying properties of the QFT fields.
- ⋆ Week 9: Came up with an intuitive formula for a QFT "state-function" and wave-functional, using the analogy/model of quantum oscillators at each point in space. The wave-functional of a field φ is the "continuous product" of the "state-function" probability amplitudes $\Psi(\mathbf{x}, \phi)$ over that field:

$$\Psi[\varphi] = \prod_{\mathbf{x}} \Psi\big(\mathbf{x}, \varphi(\mathbf{x})\big) = \prod_{\mathbf{x}} e^{\ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)} = e^{\sum_{\mathbf{x}} \ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)} = e^{\int \mathrm{d}^3x \, \ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)}$$

  Using this formula with ground wave-functions of uncoupled oscillators yields an expression (on left) very similar to the expression given in Hatfield (on right):

$$\Psi_0[\varphi] \propto e^{-\frac{1}{4} \int \mathrm{d}^3x \, \varphi^2(\mathbf{x})/{\phi_0}^2} \qquad \Psi_0[\varphi] \propto e^{-\int \mathrm{d}^3x \, \mathrm{d}^3y \;\, \varphi(\mathbf{x}) \, g(\mathbf{x},\mathbf{y}) \, \varphi(\mathbf{y})}$$

  I have yet to figure out how to derive the expression on the right for coupled oscillators intuitively.

- ⋆ Week 10: Learned functional calculus by extensively charting the analogy between functionals and functions of finite-dimensional vectors. Explored the meaning of "second order functional derivatives" (an issue that comes up in QFT) and proposed a functional Taylor series (and a functional gradient).
- ⋆ Bonus: Explored the proper meanings of continuous products and sums. Further developed the implications of the intuitive wave-functional given above (Week 9).