Review of Winter 2008 Lecture Series

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- $\star \Rightarrow$ original material produced/proposed

- $\star$ Week 1: (Spin) Explicitly worked out what is meant by "a symmetric, traceless (2×2) tensor has a period of 180°." This was the first step in a three-week investigation of classical radiation of spin-2 and spin in general.

- Week 2: (Spin) Found how symmetric, traceless 2×2 tensors relate mathematically to spin-2 radiation deformations (as in the deformation of matter due to gravitational waves). I read a paper that Chris found, "What is Spin?" by Hans Ohanian, that says "...spin may be regarded as an angular momentum generated by a circulating flow of energy in the wave field [of a particle]" and "...the magnetic moment may be regarded as generated by a circulating flow of charge in the wave field." This, in conjunction with my knowledge of some arguments about the necessity of spin from Lorentz invariance (e.g., in the Dirac equation), made me realize that a deeper understanding of spin could come from a deeper understanding of quantum field theory (QFT). I spent week 3 tying up a loose end before moving on to study QFT.

- Week 3: (Spin) Found how symmetric, traceless 2×2 tensors relate via Einstein's field equations to spin-2 radiation deformations.

- $\star$ Week 4: (QFT) Posed the fundamental questions whose answers are my first goal in understanding QFT. Proposed an answer to "What information does a QFT state contain?" and various notations to symbolize that information, including this "Fock ket":

$$\| \Psi(t) \rangle\!\rangle = \Big(\big|\Psi_1(t)\big\rangle , \big|\Psi_2(t)\big\rangle , \ldots , \big|\Psi_n(t)\big\rangle , \ldots \Big)$$

  In courses on QFT, very little, if any, mention is made of actual states and how one might graph or plot them. That is one of our goals.

- Week 5: (QFT) Showed how one discovers particle creation and annihilation operators for the Schrödinger and Klein-Gordon equations of motion. Using Hatfield's book, we see that one "completes the square" or "manifests an operator square" in the equation of motion to find these operators. At the time, I thought I would do similar procedures for other equations of motion to find other creation and annihilation operators, but now I think that once you get them for the Klein-Gordon equation, that is it. You use these operators along with perturbation theory with other, more complicated equations of motion. I also showed how Fourier transformation relates various objects, such as $|\mathbf{x}\rangle$ and $|\mathbf{p}\rangle$.

- $\star$ Week 6: (QFT) During this week I read more than usual, so I did not have as much material for my lecture. But I did find a way to express the (eigen) wave-functional $\Psi_1[\varphi]$ (from Hatfield Eqn. 10.30) more generally in terms of wave-functions and Dirac bras and kets.

- $\star$ Week 7: (QFT) Derived the Klein-Gordon equation (and more) from a classical physical model. Not only did this help give some intuition for the equation and anything that obeys the equation, but it lead to an explanation that I've never seen in a textbook: the dispersive nature of the Klein-Gordon equation allows signals with frequency/energy greater that mc^2 to propagate freely but causes signals with lower energy to decay. I showed that the decaying signals do not seem to correlate to virtual particles.

- Week 8: (QFT) Derived the Klein-Gordon Green's function / propagator. This was in an attempt to answer a question that turned out to be too advanced for me right now: how does the Klein-Gordon Green's function / propagator relate to raising/lowering of the field (in a classical sense, in particular)? Also, I tried (mostly unsuccessfully) to find out what the general solutions to the Klein-Gordon equation are, for reference when studying properties of the QFT fields.

- ⋆ Week 9: (QFT) Came up with an intuitive formula for a QFT "state-function" and wave-functional, using the analogy/model of quantum oscillators at each point in space. The wave-functional of a field φ is the "continuous product" of the "state-function" probability amplitudes $\Psi(\mathbf{x}, \phi)$ over that field:

  $$\Psi[\varphi] = \prod_{\mathbf{x}} \Psi\big(\mathbf{x}, \varphi(\mathbf{x})\big) = \prod_{\mathbf{x}} e^{\ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)} = e^{\sum_{\mathbf{x}} \ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)} = e^{\int \mathrm{d}^3x \, \ln \Psi\left(\mathbf{x}, \varphi(\mathbf{x})\right)}$$

  Using this formula with ground wave-functions of uncoupled oscillators yields an expression (on left) very similar to the expression given in Hatfield (on right):

  $$\Psi_0[\varphi] \propto e^{-\frac{1}{4} \int \mathrm{d}^3x \, \varphi^2(\mathbf{x}) / {\phi_0}^2} \qquad \Psi_0[\varphi] \propto e^{-\int \mathrm{d}^3x \, \mathrm{d}^3y \;\, \varphi(\mathbf{x}) \, g(\mathbf{x}, \mathbf{y}) \, \varphi(\mathbf{y})}$$

  I have yet to figure out how to derive the expression on the right for coupled oscillators intuitively.

- ⋆ Week 10: (QFT) Learned functional calculus by extensively charting the analogy between functionals and functions of finite-dimensional vectors. Explored the meaning of "second order functional derivatives" (an issue that comes up in QFT) and proposed a functional Taylor series (and a functional gradient).

- ⋆ Bonus: (QFT) Explored the proper meanings of continuous products and sums. Further developed the implications of the intuitive wave-functional given above (Week 9).