Physics 6B: Problem 34 from Chapter 21 of Serway & Jewett

"Solving" a Circuit

34. The ammeter shown in Figure 1 below reads 2.00 A. Find I_1, I_2, and $\mathcal{E}$.

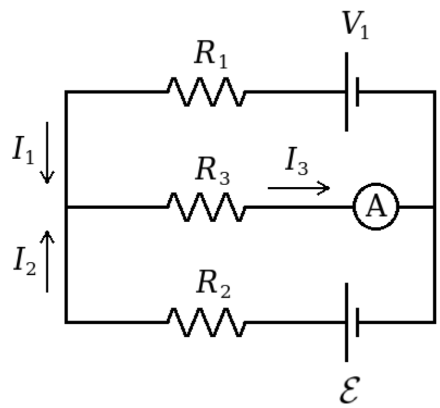

Figure 1: Circuit Diagram

Given

$V_1 = (15.0\text{ V}),\quad R_1 = (7.00\ \Omega),\quad R_2 = (2.00\ \Omega),\quad R_3 = (5.00\ \Omega),$ and $I_3 = (\pm 2.00\text{ A}).$

Initial observations

Since we don't know the orientation of the ammeter, we have that the meter reading (2.00 A) could be in either direction, so I_3 could be positive or negative. However, given the orientation of the voltage sources (which could be batteries, cells, or some other seats of emf), we can imagine that positive charges are moving in the direction of the arrows for I_1 and I_2. Then we expect that all of these positive charges will also be moving along the direction of the arrow for I_3, meaning that I_3 should have a positive value. I will do this problem without assuming that I_3 is positive and we'll see how it might be negative.

Applying Principles and Algebra

We can use Kirchhoff's circuit laws to create three linear equations and solve for these three unknowns, I_1, I_2, and $\mathcal{E}$. Equation 1 below is created using Kirchhoff's voltage law (KVL) on the top loop of the circuit, proceding counter-clockwise from the top right corner. The ammeter is assumed to be ideal, so it has negligible influence on the circuit and so has no change in electric potential across it. Equation 2 is created using KVL on the bottom loop of the circuit, proceding counter-clockwise from the top right corner of the loop. Equation 3 is created using Kirchhoff's current law (KCL) on the left junction of the circuit, taking inward current arrows as positive and outward current arrows as negative.

$$V_1 - I_1R_1 - I_3R_3 + 0 = 0 \tag{1}$$

$$0 + I_3R_3 + I_2R_2 - \mathcal{E} = 0 \tag{2}$$

$$I_1 + I_2 - I_3 = 0 \tag{3}$$

Now, using these equations in the order 1, 3, 2, we can solve for the unknowns:

$$I_1 = \frac{V_1 - I_3 R_3}{R_1} = \frac{(15.0\ \text{V}) - (\pm 2.00\ \text{A})(5.00\ \Omega)}{(7.00\ \Omega)} = \left\{ \left(\frac{5}{7}\ \text{A}\right) \text{ or } \left(\frac{25}{7}\ \text{A}\right) \right\}$$

$$I_2 = I_3 - I_1 = (\pm 2.00\ \text{A}) - \big\{(5/7\ \text{A}) \text{ or } (25/7\ \text{A})\big\} = \left\{ \left(\frac{9}{7}\ \text{A}\right) \text{ or } \left(-\frac{39}{7}\ \text{A}\right) \right\}$$

$$\begin{aligned} \mathcal{E} = I_3 R_3 + I_2 R_2 &= (\pm 2.00\ \text{A})(5.00\ \Omega) + \big\{(9/7\ \text{A}) \text{ or } (-39/7\ \text{A})\big\}(2.00\ \Omega) \\ &= \left\{ \left(\frac{88}{7}\ \text{V}\right) \text{ or } \left(-\frac{148}{7}\ \text{V}\right) \right\} \end{aligned}$$

So, we have two options for our full solution set:

$$(I_1, I_2, \mathcal{E}) = \left\{ \left(\frac{5}{7}\ \text{A}, \frac{9}{7}\ \text{A}, \frac{88}{7}\ \text{V}\right) \text{ or } \left(\frac{25}{7}\ \text{A}, -\frac{39}{7}\ \text{A}, -\frac{148}{7}\ \text{V}\right) \right\},$$

where the first option is true if I_3 is positive and the second option is true if I_3 is negative. We now see that I_3 could only be negative if $\mathcal{E}$ were negative; but we tend to expect that when someone draws the emf (or battery) symbol with a voltage rating $\mathcal{E}$, they mean for that rating to be positive. Thus we expect that the first set of solutions is the correct set.

$$I_1 = \left(\frac{5}{7}\ \text{A}\right), \quad I_2 = \left(\frac{9}{7}\ \text{A}\right), \quad \mathcal{E} = \left(\frac{88}{7}\ \text{V}\right)$$

or

$$\boxed{I_1 = (0.714\ \text{A}), \quad I_2 = (1.29\ \text{A}), \quad \mathcal{E} = (12.8\ \text{V})}$$

References

[1] Raymond A. Serway; John W. Jewett, Jr.: *Principles of Physics Vol. 2: 6B / 6C - UCLA, Fourth Edition*, Cengage Learning (2008)