Physics 1C: Problem 55 from Chapter 33 of Giancoli
Apparent Depth of a Pool of Water Covered with a Slab of Glass

55. A 12.0-cm-thick plane piece of glass ($n = 1.50$) lies on the surface of a 12.0-cm-deep pool of water. How far below the top of the glass does the bottom of the pool seem as viewed from directly above?

Initial observations

If the apparent depth of the pool is different from its actual depth, that means that the image of the bottom of the pool is in a different location from the actual bottom of the pool. This calls for a ray diagram. Now, basic depth perception comes from having more than one perspective on an object; two eyes give us two perspectives. So we should have at least two rays in our diagram that start from the bottom of the pool (the "object") and end at each of the observer's eyes. If we trace the ray segments that reach the eyes back toward the object, they should meet at a point other than the point where they actually originate. (Of course, since the bottom of the pool does not emit light, the rays are reflected from the point and do not actually originate there.)

Diagram and Calculation

We draw the diagram (Fig. 1) in an exaggerated fashion so as not to confuse ourselves in the equation-writing process. (The observer's eyes would not be so far apart in reality.) We include the angles in a separate diagram (Fig. 2) to avoid clutter.

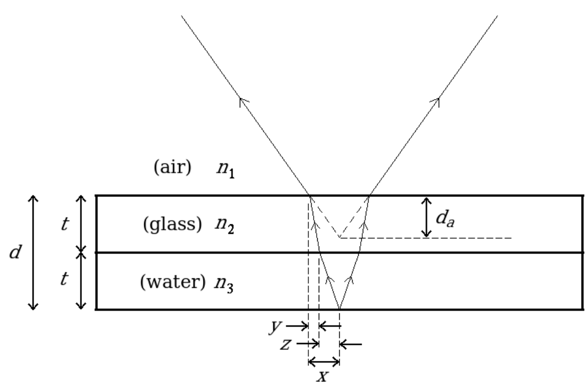

Figure 1: Exaggerated ray diagram.

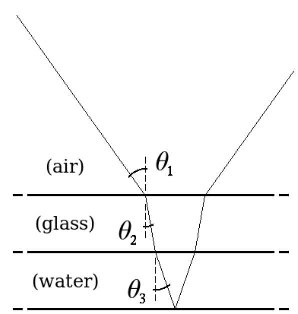

Figure 2: Ray diagram angles.

As we've labelled the diagram, $t = (12.0\text{ cm})$, $n_1 = n_{\text{air}} = 1$, $n_2 = n = 1.50$, and $n_3 = n_{\text{water}} = 1.33$. We want to solve for the apparent depth d_{a} in terms of these known quantities. Note that $d = 2t$ and $x = y + z$.

Using Snell's law, we have $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$, but since θ_1 is actually very small (and therefore the other angles are very small), we can approximate these relations as

$$n_1\theta_1 = n_2\theta_2 = n_3\theta_3.$$

We also have $\tan\theta_1 = x/d_{\text{a}}$, $\tan\theta_2 = y/t$, and $\tan\theta_3 = z/t$, which can likewise be approximated as

$$\theta_1 = \frac{x}{d_{\text{a}}} \qquad \theta_2 = \frac{y}{t} \qquad \theta_3 = \frac{z}{t}.$$

Now let's see if we can get d_a in terms of n_1, n_2, n_3, and t:

$$\begin{aligned} d_\mathrm{a} &= \frac{x}{\theta_1} = \frac{y+z}{\theta_1} = \frac{\theta_2 t + \theta_3 t}{\theta_1} = \left(\frac{\theta_2 + \theta_3}{\theta_1}\right) t \\ &= \left(\frac{\left(\frac{n_1}{n_2}\theta_1\right) + \left(\frac{n_1}{n_3}\theta_1\right)}{\theta_1}\right) t = n_1 \left(\frac{1}{n_2} + \frac{1}{n_3}\right) t \end{aligned}$$

Therefore, while the actual depth is

$$d = 2t = 2(12.0 \text{ cm}) = (24.0 \text{ cm}),$$

the apparent depth is

$$d_\mathrm{a} = n_1 \left(\frac{1}{n_2} + \frac{1}{n_3}\right) t = (1)\left(\frac{1}{1.50} + \frac{1}{1.33}\right)(12.0 \text{ cm}) = (17.0 \text{ cm}).$$

Note that we can now see that the diagram was a little off: the ray segments should trace back to a point within the water rather than the glass.

When viewed from directly above, the bottom of the pool seems to be $\boxed{17.0 \text{ cm}}$ below the top of the glass.

References

[1] Douglas C. Giancoli: *Physics for Scientists & Engineers with Modern Physics, Third Edition*, Prentice Hall (2000)