Physics 1C: Conventions for Mirror Equations and Lens Equations

Suggestion: First check out this summary:
`http://aforrester.bol.ucla.edu/educate/Articles/For_Students/MirrorLensEqnConv.pdf`
Then check out the "Alternate, Consistent Conventions" here, on the second page below.

Depending on your own tastes and brain structure, one of these sets of conventions may be more appealing to you. Use whichever one you prefer.

Giancoli's Traditional Conventions

- Giancoli uses the same equations for mirrors and lenses

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f} \qquad m \equiv \frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{d_\mathrm{i}}{d_\mathrm{o}}$$

  but different conventions in each case for the signs of quantities.

  - d represents a "distance" that may be positive or negative.
    - d_o is the distance from the mirror/lens to the object.
    - d_i is the distance from the mirror/lens to the image.
  - f represents a "distance" called the "focal length", which can be positive or negative.
    - f is the distance from the mirror/lens to the focal point.
  - h_o, the height of the object, is taken to be positive in all cases, and h_i, the "height" of the image, is positive for an upright image and negative for an inverted image (relative to the object).
  - m is the lateral magnification of the image. Due to the convention for h_i, it is positive for an upright image and negative for an inverted image (relative to the object).
- (Parabolic) Mirror Equations Conventions
  - When the object, image, or focal point is on the side of the mirror from which the light is coming (on the reflecting side), the corresponding distance is considered positive.
  - If any of these points is behind the mirror, the corresponding distance is negative.

    (So the "focal length" is positive for concave mirrors and negative for convex mirrors.)
- (Thin) Lens Equations Conventions
  - The "focal length" is positive for converging lenses and negative for diverging lenses.
  - Different conventions for object and image:
    - The object distance is positive if it is on the side of the lens from which the light is coming (this is usually the case, although when lenses are used in combination, it might not be so); otherwise, it is negative.
    - The image distance is positive if it is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. Equivalently, the image distance is positive for a real image and negative for a virtual image.

Alternate, Consistent Conventions

- We may use different equations for mirrors and lenses but the same conventions in each case. The lateral magnification is the same for each:

$$m \equiv \frac{y_{\rm i}}{y_{\rm o}} = -\frac{x_{\rm i}}{x_{\rm o}}$$

  - The focal length f is a positive quantity, always, just as anything called "length" should be. Anything called a "distance" should also be positive. We will thus refer to (relative) positions instead, which can be negative.
  - Define an x-axis along the principle axis of the mirror or lens and a y-axis perpendicular to it. Let the position of the mirror or lens be $x = 0$ and the position of the x-axis be $y = 0$. (It doesn't matter which way either of the axes points.)
    - $x_{\rm f}$ is the position of the focal point that is relevant for incoming rays that are parallel to the axis. A mirror has only one focal point, but lenses have two, one of which is relevant for these parallel rays.
    - $x_{\rm o}$ is the position of the object.
    - $x_{\rm i}$ is the position of the image.
      - These x-positions (or axial positions) are relative to the mirror or lens (which is at $x = 0$) and their signs are immediately apparent in a ray diagram once the x-axis is defined. Note that either $x_{\rm f} = +f$ or $x_{\rm f} = -f$.
    - $y_{\rm o}$ is the lateral position of a point on the object (usually the extremal lateral point, which would give the "height" using Giancoli's conventions if one side of the object is on the axis).
    - $y_{\rm i}$ is the lateral position of the point in the image corresponding to the point on the object chosen for $y_{\rm o}$.
      - These y-positions (or lateral positions) are relative to the axis (which is at $y = 0$) and their signs are immediately apparent in a ray diagram once the y-axis is defined.
- (Parabolic) Mirror Equation[1]

$$\frac{1}{x_{\rm i}} = \frac{1}{x_{\rm f}} - \frac{1}{x_{\rm o}}$$

- (Thin) Lens Equation and Convention

$$\frac{1}{x_{\rm i}} = \frac{1}{x_{\rm f}} + \frac{1}{x_{\rm o}}$$

  - For lenses there are two focal points, each of which are equally relevant in a ray diagram. As stated above, by convention we choose to use the focal point that is relevant for incoming rays that are parallel to the axis:
    - for converging lenses, the focal point on the side to which the light is going;
    - for diverging lenses, the focal point on the side from which the light is coming.

[1]The equation is organized so that the quantities you would naïvely expect to know are on the right, so you can plug in known values and find the unknown quantity on the left. Of course, in a given problem the knowns and unknowns can be different.

Relation of Conventions

The traditional conventions that Giancoli uses are related to the alternate conventions by

- (for a mirror) choosing the x-axis to point to the side from which the light is coming (toward the reflecting side of the mirror);
- (for a lens) choosing an x-axis to point to the side
  - to which the light is going for the focal point and image,
  - from which the light is coming for the object;
- (for a mirror or lens) choosing the y-axis to point in the "upward" direction of the object, so y_{o} is positive.

References

[1] Douglas C. Giancoli: *Physics for Scientists & Engineers with Modern Physics, Third Edition*, Prentice Hall (2000)