Notes for Physics 1B

Dipole Potential Energy

1 Torque

Given that an electric dipole with electric dipole moment $\mathbf{p}$ in an electric field $\mathbf{E}$ experiences a torque $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$, an electric dipole is forced to align with the electric field. If there is no dissipation of energy, the dipole will oscillate towards alignment, overshoot, be forced back, and continue oscillating like this forever. If there is dissipation of energy, the dipole will settle down in damped oscillation to align with the electric field.

The same goes for a magnetic dipole of magnetic dipole moment $\boldsymbol{\mu}$ in a magnetic field $\mathbf{B}$: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.

2 Dipole Potential Energy

Potential energy U (with respect to a reference position $\mathbf{r}_0$) is defined using a conservative force $\mathbf{F}_\mathrm{c}$:

$$U(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F}_\mathrm{c} \cdot \mathrm{d}\mathbf{r}'.$$

This is the work that must be done against the conservative force in moving from $\mathbf{r}_0$ to $\mathbf{r}$. This can be generalized to conservative torques:

$$U(\theta) = -\int_{\theta_0}^{\theta} \tau_z \, \mathrm{d}\theta,$$

where τ_z and θ are about the same z-axis.

Given that the direction of $\mathbf{E}$ is constant and θ is the angle between $\mathbf{p}$ and $\mathbf{E}$, measured from $\mathbf{E}$, we have $\tau_z = -pE\sin\theta$,

$$U(\theta) = -\int_{\theta_0}^{\theta} \tau_z \, \mathrm{d}\theta' = \int_{\theta_0}^{\theta} pE\sin\theta' \, \mathrm{d}\theta' = pE\left[-\cos\theta'\right]_{\theta_0}^{\theta} = -pE\left(\cos\theta - \cos\theta_0\right).$$

If we select the reference angle to be $\theta_0 = \pi/2$, where $\mathbf{p}$ is perpendicular to $\mathbf{E}$, then

$$U(\theta) = -pE\cos\theta = -\mathbf{p} \cdot \mathbf{E}.$$

The same argument holds for magnetic dipoles:

$$U(\theta) = -\mu B\cos\theta = -\boldsymbol{\mu} \cdot \mathbf{B}.$$