

Some Traditional Conventions for the Mirror, Interface, and Lens Equations

The following optics equations are “small-angle” equations because they are derived assuming that the spherical surface (of the mirror, interface, or lens) is a small portion of the total sphere that it delimits, so the angle θ between the central axis and the line connecting the center of curvature to the outer edge of the spherical surface is small enough that $\sin \theta \approx \theta$, within the precision of measurements in the problem. With that assumption, the equations can be derived with ray diagrams using Snell’s law of refraction and geometry (similar triangles and some trigonometry).

| “Small-Angle” Optics Equations | | |
|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Shallow Spherical Mirror (reflection) | Shallow Spherical Interface (single refraction) | Thin Lens (with spherical sides) (double refraction) |
| $f = \frac{1}{2}R$ $\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ $m \equiv \frac{y_i}{y_o} = -\frac{i}{o}$ | $f = \left(\frac{n_2}{n_2 - n_1} \right) R$ $\frac{n_2 - n_1}{R} = \frac{n_1}{o} + \frac{n_2}{i}$ $?$ | $\frac{1}{f} = \left(\frac{n}{n_m} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ $\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ $m \equiv \frac{y_i}{y_o} = -\frac{i}{o}$ |

R = radius of curvature. f = “focal length” or focal position. n_1 = refraction index of medium that light comes from. n_2 = refraction index of medium that light goes to. n = refraction index of lens. n_m = refraction index of medium in which lens is placed. R_1 = radius of curvature of one side of the lens. R_2 = radius of curvature of the other side of the lens. o = object position. i = image position. m = lateral magnification. y_o = “object height” or

lateral position of a certain point of the object. y_i = “image height” or lateral position of the corresponding point of the image. Axial positions are measured from the mirror/interface/lens surface. Lateral positions are measured from the central axis.

| Orientation Conventions | |
|-------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| o | The object position is measured in the direction against which the light is coming. |
| i | The image position is measured in the direction that the light leaves the surface. |
| f | The (main or only) focal position is measured in the direction that the light leaves the surface. For lenses, a second focus is at $-f$. |

| Sign Conventions | | | |
|------------------|-------------------------------------------------------------------------------------------------------------------|----------------|---------------------------------------------------------------------------|
| + | converging (mirror/interface/lens) | $f > 0$ | rays get bent together: <i>concave</i> mirror, <i>biconvex</i> lens, etc. |
| - | diverging (mirror/interface/lens) | $f < 0$ | rays get bent apart: <i>convex</i> mirror, <i>biconcave</i> lens, etc. |
| + | real (image/object) | i or $o > 0$ | image/object on side light (goes to)/(comes from) |
| - | virtual (image/object) | i or $o < 0$ | image/object on side light (comes from)/(goes to) |
| + | non-inverted (image) | $m > 0$ | image has same orientation as object |
| - | inverted (image) | $m < 0$ | image has opposite (flipped) orientation |
| \pm | For mirrors and lenses, R has the same sign as f . (+ converging; - diverging) | | |
| + | For interfaces, R is positive if the surface is convex from the perspective of the incoming light. (+ convex) | | |
| - | For interfaces, R is negative if the surface is concave from the perspective of the incoming light. (- concave) | | |

Exercises: Solve for the focal position of a lens where $R_1 = R_2 \equiv R$. Given a thin lens with R_1 and R_2 , solve for f_1 and f_2 using the interface formula and show how f relates to f_1 and f_2 . Use the interface equations to derive the thin lens equations. Derive the equations above.