Conductivities in Bilayer Quantum Hall Systems

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Abstract

We use a computer program to make correlations between various parameters of a bilayer quantum Hall system. We first introduce the regular, classical Hall effect, the quantum Hall effect, and the bilayer quantum Hall system. Then we describe the information gathered with the program. From the original work and data, we conclude that the bilayer conductivities σ_{00} and σ_{zz} are fully dependent on the order parameter M_0, where $\sigma_{00} = \sigma_L - \sigma_{L2}$ and $\sigma_{zz} = \sigma_L + \sigma_{L2}$. ($\sigma_L$ is the longitudinal conductance within a single layer and σ_{L2} is the longitudinal conductance between layers.) The program is determined to be accurate and useful, and we conclude that we can model the experimental bilayer transition of large d/l to small d/l by changing the order parameter (which varies with disorder and filling factor), as was done with the program. (d is the distance between the layers and l is the magnetic length.)

1 Introduction

The quantum Hall effect (QHE) is an intriguing macroscopic phenomenon involving dissipationless current and the quantization of Hall conductance. It occurs in two-dimensional electron systems at very low temperatures ($T \approx$ 5mK) and in the presence of strong perpendicular magnetic fields ($B \approx$ 5-20 Tesla). As a topic in solid state physics, it has been spurring a great deal of experimental and theoretical research in the past quarter century and continues to provide material for new theories and mathematical applications. Aside from the intellectual challenges and pleasures it supplies, its practical applications merit the attention it has received. The values of the quantized Hall conductance are so accurate as integer multiples of e^2/h (by about one part in 10^8) that they have become used in international standards for resistance. (e is the electron charge and h is Planck's constant.) Conveniently, the fine-structure constant, $\alpha = e^2/\hbar c$ (in cgs units), which measures the strength with which subatomic particles interact with each other and with light, is measured relatively easily using this technique as well, since c is defined exactly. And since the two-dimensional systems under study are realized with semiconductor heterojunctions, the QHE has importance for the computing industry in its quest to know everything about semiconductors.

There are actually two types of quantum Hall effects: the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE). On the surface, the only noticeable differences between the two effects are that the FQHE results in fractional rather than integer multiples of e^2/h for Hall conductance and requires a stronger magnetic field, lower temperatures, and a sample with fewer impurities. However, the theoretical explanations for how each effect arises from physical laws are quite different. We deal only with the integer QHE in this paper. In fact, we focus on a special case of the QHE in which two parallel, two-dimensional electron gases (2DEGs) are situated close together, instead of the ordinary case of just one 2DEG. This case is interesting because of the quantum behavior of the electrons as they inhabit and interact in both of the layers.

The original work that we have done on this topic involves the use of a program (written by Yogesh Joglekar in C) to model the bilayer quantum Hall system and calculate relationships among various parameters, particularly in regions that are more difficult to evaluate analytically. So the purpose of this paper is to introduce the bilayer quantum Hall system and describe the information gathered with the program. Thus there are two main sections

entitled *The Theory* and *The Program*.

2 The Theory

2.1 Classical Hall Effect

So, how does a bilayer quantum Hall system work? Before answering that, a little background is in order. Before the quantum Hall effect, there came the ordinary, classical Hall effect, which was discovered by Edwin Hall in 1879. In his investigation, Hall applied a voltage V_L (see Figure 1.) across a thin, flat ribbon of conducting material such that a current I was directed lengthwise, or longitudinally through the ribbon. He placed the ribbon in a magnetic field B oriented perpendicularly to its flat surface. With this situation, the charge carriers (electrons) are deflected by the Lorentz force F_B, and they begin to create a potential difference transversely across the conductor. The transverse electric field associated with this potential difference also creates a force F_E that acts on the electrons and quickly grows to equilibrate with the Lorentz force, allowing electrons to flow without being deflected. This behavior of the electrons under these conditions is called the Hall effect[1]. The equilibrated transverse potential difference is called the Hall voltage V_H, and the Hall resistance R_H is defined by $V_H = IR_H$. Hall conductance G_H is defined as $1/R_H$, and the longitudinal resistance R_L is defined by $V_L = IR_L$.

The mathematics describing this equilibrium situation can be represented by vectors and matrices:

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \rho\vec{j} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix}, \tag{1}$$

where $\vec{E}$ is the electric field vector, ρ is the resistivity tensor, and $\vec{j}$ is the current density vector. What is measured in experiments is not the electric field or the resistivities, but rather the voltages and resistances. However, since we deal with uniform conductors, the resistivities and electric fields are related to the measurables by simple multiplicative factors, which are determined by the dimensions of the ribbon. (If l is the length of the ribbon, w is the width, t is the thickness, and $A = wt$ is the cross-sectional area looking

[1]Hall's discovery was inspired by the mistaken claim James Clerk Maxwell made that "the mechanical force which urges a conductor ... acts, not on the electric current, but on the conductor which carries it."[1]

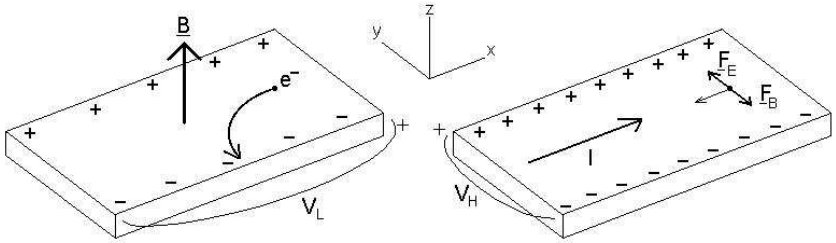

Figure 1: Setup for the classical Hall effect. With the system not yet in equilibrium (left), electrons are deflected. In equilibrium (right), there is no transverse current and the Hall voltage is attained. (Although not shown on the right, the magnetic field and longitudinal voltage are still applied.)

longitudinally at the ribbon, then $E_x = V_L/l$, $E_y = V_H/w$, $\rho_{xx} = R_L/Al$, and $\rho_{yx} = R_H/Aw$.)

All samples we shall consider are essentially uniform and rotationally invariant, so if we apply a voltage in the x direction and measure the current in that direction, we would expect the same result if we turned the sample and did likewise in the y direction. Thus $\rho_{xx} = \rho_{yy}$. If we see an effect in the y direction due to voltage applied in the x direction, we would also expect a voltage applied in the y direction would produce the same effect in the negative x direction. Therefore, the resistivity tensor becomes

$$\rho = \begin{bmatrix} \rho_{xx} & -\rho_{yx} \\ \rho_{yx} & \rho_{xx} \end{bmatrix}. \tag{2}$$

Since the transverse current, and thus j_y, is zero when the system is in equilibrium, equation (1) simplifies to

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \rho_{xx} j_x \\ \rho_{yx} j_x \end{bmatrix}. \tag{3}$$

This is analogous to the equations mentioned earlier defining longitudinal and Hall resistances, so we shall use the same names and subscripts for the resistivities and electric field components: $\rho_{xx} = \rho_L$, $\rho_{yx} = \rho_H$, $E_x = E_L$, and $E_y = E_H$. Furthermore, summing the magnetic and electric forces to get a net transverse force of zero on a given electron shows that $E_y = v_x B = B j_x/(nec)$ (in cgs units), where v_x is the longitudinal velocity of the electrons, n is the

areal density of electrons, e is the charge of the electron, and c is the speed of light. Comparing this with the previous equation (specifically, $E_y = \rho_H j_x$) leads to the conclusion that ρ_H, and thus R_H, is proportional to B. ($R_H = (Aw/nec)B$.) We shall see how this changes in the QHE case.

2.2 Quantum Hall Effect

Now, an introduction to the regular monolayer quantum Hall effect should occur before analyzing the bilayer configuration. The QHE was discovered by Klaus von Klitzing in 1980[2] [3]. He found that the setup for the Hall effect, when put to extremes, held unexpected and interesting characteristics. When the electrons are restricted to a conducting "ribbon" that is about 10 nm thick, when the magnetic field's strength is increased to about 5 Tesla or more, and when the system's temperature is lowered to about 1 Kelvin or less, the Hall conductivity is observed to take on values that are extremely precise integer multiples of e^2/h and the longitudinal conductivity becomes essentially zero. Experimental data can be seen in Figure 2 [4], on page 13.

As mentioned in the introduction, a semiconductor heterojunction provides the "ribbon" in which the electrons can flow. A heterojunction is simply the interface of two different semiconductors, and the potential energy dip of the junction creates a thin "quantum well" situation. With the temperature so low, the electrons do not have enough energy to escape this well. Even though the layer that the electrons inhabit and freely move around in does have some thickness, it can effectively be considered a 2-D system containing a 2-D electron gas (2DEG). And with this low dimensionality comes a new relation between resistance and resistivity. Since $R = \rho L^{(2-d)}$ for a conductor of d dimensions and side length L, amazingly, $R = \rho$ for a 2-D L×L system. The same goes for conductance and conductivity. In actuality, the interface does not even have to be square for this relation to hold.

To consider the mathematical representation, since it's the conductivity that is quantized, we shall start with an equation containing conductivity rather than resistivity:

$$\vec{j} = \begin{bmatrix} j_x \\ j_y \end{bmatrix} = \sigma \vec{E} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}, \tag{4}$$

[2]February 5th, 2 a.m. [2]

where σ is the conductivity tensor. Applying the same arguments of symmetry as in the classical case,

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{bmatrix}. \tag{5}$$

If x is the longitudinal direction, then σ_{xx}, which relates the longitudinal electric field to the longitudinal current density, can be called the longitudinal conductivity. In the QHE regime, then, $\sigma_{xx}=0$, and

$$\begin{aligned} \rho = \sigma^{-1} = & \begin{bmatrix} 0 & \sigma_{xy} \\ -\sigma_{xy} & 0 \end{bmatrix}^{-1} = \left(\frac{1}{{\sigma_{xy}}^2}\right) \begin{bmatrix} 0 & -\sigma_{xy} \\ \sigma_{xy} & 0 \end{bmatrix} \\ = & \begin{bmatrix} 0 & -1/\sigma_{xy} \\ 1/\sigma_{xy} & 0 \end{bmatrix} = \begin{bmatrix} \rho_L & -\rho_H \\ \rho_H & \rho_L \end{bmatrix}. \end{aligned} \tag{6}$$

Here, σ_{xy} (or G_{xy}) is the inverse of ρ_H (or R_H), so there is logical justification in calling it the Hall conductivity[3] σ_H (or G_H). So, when $\sigma_L = 0$, $\rho_L = 0$, $\sigma_H = ne^2/h$, and $\rho_H = h/ne^2$, where n is an integer. (Notice that $\sigma_L=0$ *and* $\rho_L=0$.)

Looking at the QHE equivalent of equation (3), and remembering that $j_y=0$,

$$\vec{E} = \begin{bmatrix} E_L \\ E_H \end{bmatrix} = \rho\vec{j} = \begin{bmatrix} 0 & -\frac{h}{ne^2} \\ \frac{h}{ne^2} & 0 \end{bmatrix} \begin{bmatrix} j_x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \left(\frac{h}{ne^2}\right) j_x \end{bmatrix}, \tag{7}$$

and we see that the longitudinal electric field is zero even while there is a longitudinal current. This is the dissipationless current that was mentioned in the introduction[4]. (Since the current is generated by a voltage V_L applied across the 2DEG, there must be a change in potential somewhere, and that change occurs at the edges of the sample.) That's enough of the monolayer QHE; now onto the bilayer configuration.

[3]Due to the manner in which Hall resistance and conductance were defined in the classical Hall effect, the relationship of Hall resistivity to σ_{xy} is not as simple as for the QHE, so there is no conductivity that could meaningfully be called Hall conductivity.

[4]Even though there is no resistance, and it looks like a supercurrent, this process is not technically superconducting because of the presence of the magnetic field, among other reasons.

2.3 Bilayer Configuration

The mathematics of the bilayer situation is a little more complicated. Since there are two layers (called "Top" and "Bottom"), not only are there interactions of electric field and current density within each layer (Top-Top and Bottom-Bottom interactions), there are also interactions of the electric field in one layer with the current density in the other layer (Top-Bottom and Bottom-Top interactions). So there are four times as many elements in the bilayer conductivity matrix as in the monolayer matrix:

$$\sigma = \begin{bmatrix} \sigma_{xx}^{TT} & \sigma_{xy}^{TT} & \sigma_{xx}^{TB} & \sigma_{xy}^{TB} \\ \sigma_{yx}^{TT} & \sigma_{yy}^{TT} & \sigma_{yx}^{TB} & \sigma_{yy}^{TB} \\ \sigma_{xx}^{BT} & \sigma_{xy}^{BT} & \sigma_{xx}^{BB} & \sigma_{xy}^{BB} \\ \sigma_{yx}^{BT} & \sigma_{yy}^{BT} & \sigma_{yx}^{BB} & \sigma_{yy}^{BB} \end{bmatrix}. \tag{8}$$

We have already described, named, and used symmetry arguments for the same-layer conductivities. Now, by analogy with the same-layer conductivities, σ_{xx}^{TB} is named σ_{L2} and σ_{xy}^{TB} is named σ_{H2}. Using the same symmetry arguments as before, $\sigma_{yy}^{TB} = \sigma_{xx}^{TB}$ and $\sigma_{yx}^{TB} = -\sigma_{xx}^{TB}$. And by applying an argument for inversion invariance (looking at the symmetries if the sample is flipped over), we conclude that

$$\sigma = \begin{bmatrix} \sigma_L & \sigma_H & \sigma_{L2} & \sigma_{H2} \\ -\sigma_H & \sigma_L & -\sigma_{H2} & \sigma_{L2} \\ \sigma_{L2} & -\sigma_{H2} & \sigma_L & \sigma_H \\ \sigma_{H2} & \sigma_{L2} & -\sigma_H & \sigma_L \end{bmatrix}. \tag{9}$$

Later, when describing the research results from the computer program, we will investigate the behavior of σ_L and σ_{L2} in the form of $\sigma_{00} = \sigma_L - \sigma_{L2}$ and $\sigma_{zz} = \sigma_L + \sigma_{L2}$. Before describing their behavior theoretically, though, a few more concepts should be covered.

An additional complexity of the bilayer configuration is that, if the conditions are right, a substantial number of electrons in each layer can tunnel into the other layer and inhabit both layers. The situation is like an infinite quantum well with a finite barrier in the center. The energy of each electron is lower than the potential barrier, so solving Schrödinger's equation yields

two types of wavefunctions: symmetric and antisymmetric. The symmetric state is of lower energy than the antisymmetric state.

A couple of parameters relate to the existence of these two states, and they are the filling factor (or filling fraction) ν and the order parameter M_0. The filling factor is the areal density of the electrons divided by the areal density of magnetic flux quanta. If ν_s is the filling factor for electrons in the symmetric state and ν_a is the same for the antisymmetric electrons, then ν can be expressed as $\nu = \nu_s + \nu_a$. The equation for the order parameter is then $M_0 = (\nu_s - \nu_a)/(\nu)$. Another expression for ν is the sum of the filling factors of the bottom and top layers, or $\nu_B + \nu_T$.

For a monolayer system, in which case ν does not relate to symmetric/antisymmetric or top/bottom states, $\nu = 1$ correlates with the presence of the QHE but $\nu = 1/2$ does not. An interesting thing happens in the bilayer system, though, when we look at the $\nu_B = \nu_T = 1/2$ case: With the layers far enough apart, the system acts the same as a two monolayers and neither of the layers show the QHE, but when the layers are very close, the same conditions bring on the QHE. So, when these two decoupled systems with dissipating current are brought close enough together, they couple and both show no dissipation! It is the whole bilayer system that shows the necessary condition of $\nu = 1$ ($\nu_B + \nu_T = 1$), but there is a transition that occurs somewhere between "far enough" and "very close." This transition is what we studied with the computer program.

Experimentally, the "close-ness" of the layers is measured by the fraction d/l, where d is the distance between the layers and l is the magnetic length[5], or, essentially, the distance between each electron. As far as the electrons are concerned, decreasing d/l is like bringing the layers closer together. This is achieved by decreasing the strength of the magnetic field, which increases l.

Now, according to theory, which is not described here, the transition from decoupled layers with no QHE to coupled layers showing no dissipation really only depends on the value of the order parameter. So the conductivities should be a function of the order parameter, and the transition can be observed by watching this relation. That is the approach taken with the use of the program. Theoretically, it is easy to determine how the conductivities should behave at the ends of the transition. At $M_0 = 0$, with no QHE,

[5]The magnetic length $l = \sqrt{\hbar c/eB}$, and $2\pi l^2$ is the area containing one magnetic flux quantum.

(or equivalently, with the layers far apart) there is essentially no interaction between the layers and σ_{L2}, which measures trans-layer interaction, goes to zero. At $M_0 = 1$, with the QHE, (or the layers very close – so close as to be nearly overlapping) the layers cannot be seen as separate and σ_L equals σ_{L2}, and $\sigma_L = 0$. Given that $\sigma_{00} = \sigma_L - \sigma_{L2}$ and $\sigma_{zz} = \sigma_L + \sigma_{L2}$, this means that at $M_0 = 0$, $\sigma_{00} = \sigma_{zz}$, and at $M_0 = 1$, $\sigma_{00} = \sigma_{zz} = 0$. Between the extremes of $M_0 = 0$ and $M_0 = 1$, though, it is not so easy to determine how σ_L and σ_{L2} (or σ_{00} and σ_{zz}) behave. That is where the program comes in.

Several more variables should be addressed before moving on to a discussion of the program. If the electric field is caused to oscillate, then it has a temporal frequency ω and a spatial frequency q. The temporal frequency is determined by the period of time it takes for the field to make one full cycle at a point in space ($\omega = 2\pi/T$, where T is the period), and the spatial frequency is determined by the distance it takes for the field to make one cycle at an instant in time ($q = 2\pi/\lambda$, where λ is the wavelength). These frequencies can be related to the susceptibility χ, and the relation will be covered in the description of the program. Also, the semiconductor sample used to create the 2DEG has a characteristic disorder. It can be quantified as a disorder within each ($s =$"same") layer v_s and a disorder that is correlated between the two ($d =$"different") layers v_d. Experimentally, these separate disorders can be controlled to a certain degree by growing the semiconductor essentially one atomic layer at a time by molecular beam epitaxy (MEB), and by controlling the placement of the doping atoms (impurities) in the semiconductors. If the impurities are placed in between the two layers, then the disorder will be correlated ($v_s = v_d$), and if the impurities are placed outside the layers, the disorder will be less correlated ($v_d \approx 0$).

3 The Program

3.1 Description

The code that was written to model the bilayer quantum Hall system essentially takes various values for parameters from the user and, using incremented values for the spatial frequency of the electric field q, calculates the associated values for the susceptibility χ, filling factor ν, and order parameter M_0. (Actually, the program returns three different values for the imaginary component of χ, but there will be more on that later.) How the

program does these calculations will not be discussed because that is beyond the scope of this paper. We will just consider it a black box. The parameters that the user sets are the incrementation of q, the frequency of the electric field ω, the disorders of the layers v_s and v_d, and the distance between the two layers, expressed as d/l. (Since l involves the strength of the magnetic field, it brings that variable into account.) While the QHE is usually studied experimentally by watching the transition from no QHE ($M_0 = 0$) to its full effect ($M_0 = 1$), the program moves in the opposite direction and changes parameters to decouple the layers.

Program Input	Program Output
q, ω, d/l, v_s, and v_d	Im χ_{00}, Im χ_{002}, Im χ_{zz}, ν, and M_0

3.2 Data Analysis

Earlier, it was noted that the behavior of $\sigma_{00} = \sigma_L - \sigma_{L2}$ and $\sigma_{zz} = \sigma_L + \sigma_{L2}$ would be analyzed in the results from the computer program, but as of yet, we have not seen any conductivities associated with the program. A little more theory will take care of that.

Using definitions and the continuity equation that relates electron density and current density (conservation of charges in each layer), it is possible to relate susceptibility χ to conductance σ by the equation

$$\text{Im}\,\chi = \lim_{\omega \to 0,\, q \to 0} \left(\frac{\sigma}{\omega}\right) q^2, \tag{10}$$

and if q^2 is kept much smaller than ω, which is small itself, then

$$\text{Im}\,\chi \approx \left(\frac{\sigma}{\omega}\right) q^2. \tag{11}$$

Since the program returns values of χ versus q, and ω is known, values of σ can be calculated using quadratic regression. And with two types of conductivity (σ_{00} and σ_{zz}) come two types of susceptibility (χ_{00} and χ_{zz}). In addition, though, the program calculates χ_{00} two different ways (as a check), so there are a total of three returned values for susceptibility.

3.3 Results

In obtaining the values for conductivity, we found that the program gave values for susceptibility (Im χ) that fulfilled a parabolic curve relative to the

values for spatial frequency q, just as it should have, according to equation (11). This indicates that the program works well, in that aspect at least, and that the values chosen for q and ω fulfilled the condition that $q^2 \ll \omega$.

Several graphs were produced that show the relationship of the filling factor ν, the order parameter M_0, and the conductivities σ_{00} and σ_{zz}. Four of them are included here, starting on page 14, and explanations are written in the captions below the graphs.

4 Conclusion

From the original work and data, we conclude that the bilayer conductivities σ_{00} and σ_{zz} are fully dependent on the order parameter M_0, where $\sigma_{00} = \sigma_L - \sigma_{L2}$ and $\sigma_{zz} = \sigma_L + \sigma_{L2}$. The disorders and distance between the layers will affect where on the σ-versus-M_0 curve the conductivities lie, but M_0 determines the curve. This conclusion fits with the theory and verifies the usefulness of the program that was used to model the bilayer quantum Hall system. The program's approximate results for the conductivities σ_{00} and σ_{zz} versus the spatial frequency of the electric field q fit nicely with the exact equation (equation 10). The equality of σ_{00} and σ_{002} turned up, as expected, in each of the relevant graphs. Also, the behavior of σ_{00} and σ_{zz} at the limits of the transition from the presence of the QHE ($M_0 = 1$) to its absence ($M_0 = 0$) was produced accurately. These three additional facts provide further support for the approximations used in the program. In sum, we conclude that we can model the experimental bilayer transition of large d/l to small d/l by changing the order parameter (which varies with disorder and filling factor), as was done with the program.

5 Acknowledgements

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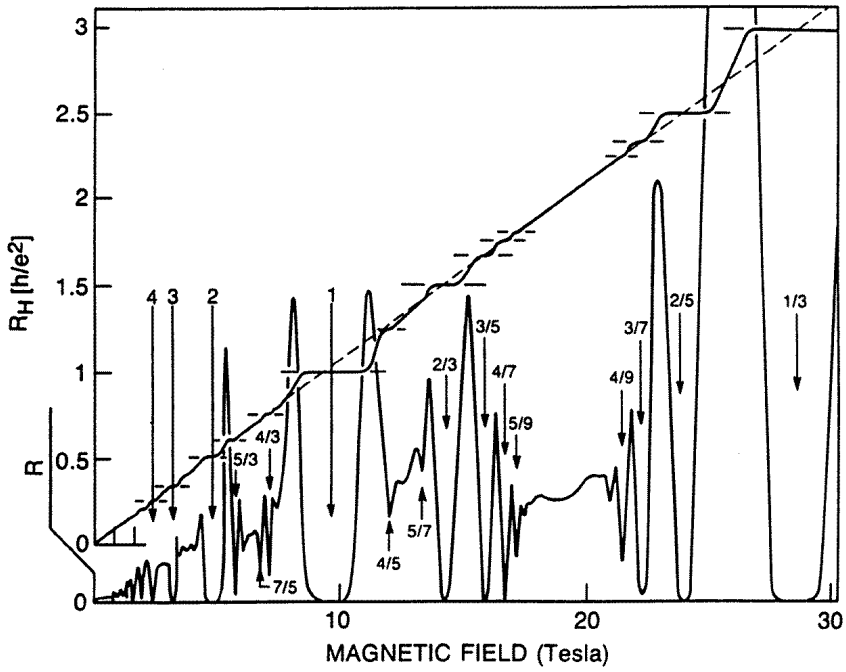

Figure 2: Hall resistance R_H (essentially the reciprocal of the Hall conductance) and dissipative longitudinal resistance R versus magnetic field strength. This graph includes fractional quantum Hall effects, and the numbers indicate the Landau level filling factors at which various features occur, so this graph is a bit more complicated than necessary at this point. Notice, though, how the relationship of Hall resistance to magnetic field is no longer linear, as was the case in the classical Hall effect. Between transitions, the Hall resistance reaches plateaus ($\frac{h}{ne^2}$, where n is an integer) while longitudinal resistance goes to zero. [4]

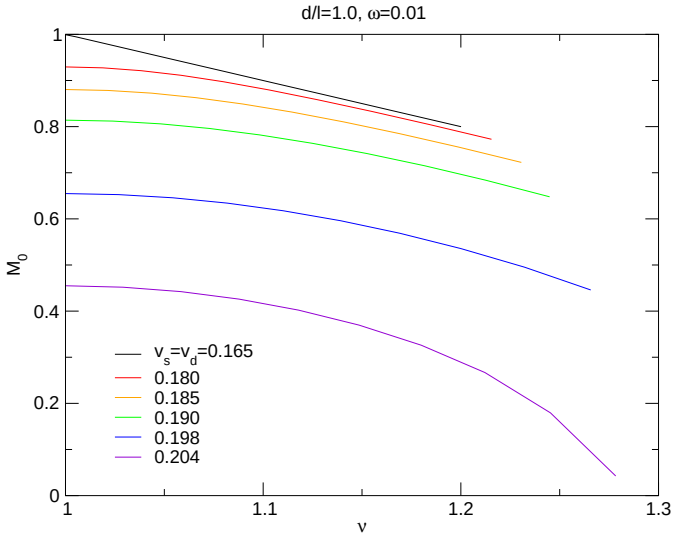

Figure 3: Order parameter M_0 versus the filling factor ν with correlated ($v_s = v_d$) disorder. Given that ν starts at one, with all the electrons in the symmetric state (ν_s=1), and increases with electrons entering the antisymmetric state, the curves behave just like we would expect, keeping the definitions of ν and M_0 in mind. As the disorders increase, more electrons move to the antisymmetric state from the symmetric state.

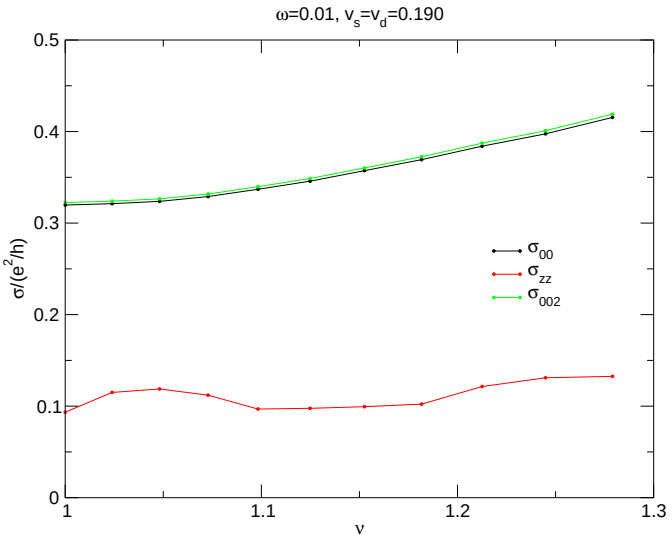

Figure 4: σ_{00} and σ_{zz} versus the filling factor ν, with correlated disorder. This is a typical graph, with one particular value of disorder.

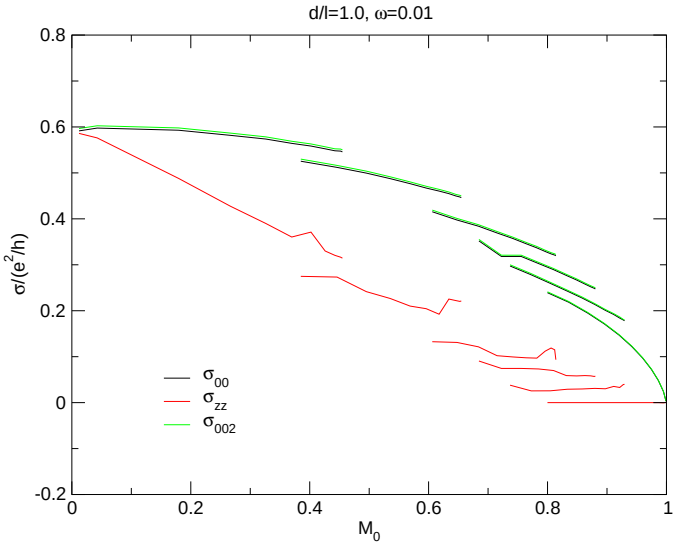

Figure 5: σ_{00} and σ_{zz} versus the order parameter M_0, with correlated disorder. Each segment of the graph is produced with a different value for disorder (while keeping v_s=v_d). The near-overlapping of the segments shows that the values for sigma are essentially dependent upon the value of M_0 only, as was predicted in theory. The values of disorder used, from left to right, are v_s=v_d = 0.204, 0.198, 0.190, 0.185, 0.180, and 0.165.

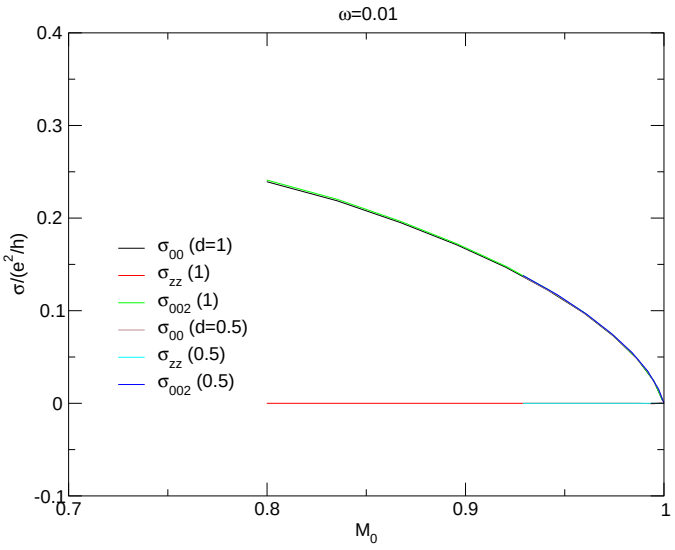

Figure 6: σ_{00} and σ_{zz} versus the order parameter M_0, with correlated disorder. Two different values for disorder (with v_s=v_d for each) and the distance d/l were used to obtain the two sets of graphs. This graph indicates strongly that the values for conductance are fully dependent on M_0. The disorders and distance between the layers will affect where on the curve the conductivities lie, but M_0 determines the curve. For d/l=1.0, the disorder is v_s=v_d= 0.165, and for d/l=0.5, the disorder is v_s=v_d= 0.204.