Simulating Quench Signals in the LHC Superconducting Dipole Magnets

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***Abstract*— A simple model was developed to describe the propagation of a quench, or quench front, and its measurement by a tool called the Local Quench Antenna (LQA). Specifically, the model addresses steady-state quench propagation that has reached a constant velocity. The quenches of interest occur in the low-temperature superconducting cables that are used in the dipole electromagnets of the Large Hadron Collider (LHC). The LQA produces a voltage signal called a "quench signal" in response to the spread of the normal-conducting region in the cables, thus the signal carries information about the quenching process. Using $Mathematica^{®}$, a program based on the model was written to simulate these signals for the purpose of studying the characteristics of a quench and determining the spatial origin of a quench in the cables. The model successfully meets these goals.**

I. Introduction

An issue that should be addressed in the introduction is what a quench is in relation to the LHC superconducting dipole magnets. First, though, this problem should be put into context.

A. Background

The scientists at CERN have decided to push the limits of experimental high-energy physics once more by creating the LHC, or Large Hadron Collider. They will accelerate two beams of protons to energies of 7-TeV each, allow the protons in the beams to collide and "explode," and thereby, with the aid of particle detectors and our current mathematical description of particle interactions, probe Nature at the smallest scales achievable today. These TeV energy levels are an order of magnitude greater than energies previously attained with the LEP/LEPII experiments and are high enough to give theorists the data they need to either move forward with their latest models (including the Higgs boson) or change their direction.

To save on costs, existing accelerator infrastructure at CERN will be re-used, and the new accelerator will jointly occupy the 27-km LEP accelerator tunnel. To keep the 7-TeV protons circulating in this particular path, there will be a need for very strong magnetic dipole fields. Specifically, since the plan is to place dipole fields over 65% of the tunnel, the magnetic fields must be 8.34 T in magnitude. To achieve such high magnitudes, the decision was made to utilize low-temperature superconducting technology; thus, the LHC superconducting dipole magnets were designed.

Within the 15-meter-long dipole electromagnets, large currents of approximately 12 kA are carried by cables made with Niobium-Titanium (NbTi), a low-temperature superconducting metal, and copper (Cu). A liquid-helium (He_3 and He_4) cryogenic system cools the cables to about 2 K, keeping their temperature below T_C, the critical temperature for superconductivity with the given current densities and magnetic fields. However, there can be a problem when, for some reason, a piece of a cable receives enough heat to put its temperature above T_C: A chain reaction is initiated that causes all of the superconducting cable to become normal-conducting and unable to support large amounts of current for very long. The strong magnetic fields are then lost, among other problems. This process of the cables becoming normal-conducting is referred to as quenching, or a quench of the magnet. The boundary between the superconducting and normal-conducting zones is called the quench front, and its movement along the cable is referred to as quench propagation.

The possible sources for the heat absorption that precipitates a quench include particle interactions, which result from protons flying off course in the magnet, and friction between the cables, which results from mechanical instability in the placement of the cables. We shall focus on the latter source. As the cables carry high currents and are situated in strong magnetic fields, they experience large forces. It is, therefore, difficult to construct the dipoles in such a way that the cables are rigidly held in place. When the newly-constructed dipoles are brought in for testing, they are brought as close to 9 T as possible until they either reach 9 T, reach 12 kA, or quench. Usually a magnet will quench before reaching the specified field strength or current value. However, a positive result is that when a magnet quenches, it usually causes the cables to shift into a more stable position, and the magnet can henceforth produce stronger fields before quenching again. The process of quenching a magnet until it performs at the targeted level is called quench training, or training the magnet.

It is desirable for physicists to receive magnets that do not require any training or, at least, that require very little training (one or two quenches) before reaching the target performance. This is because quenching can put a lot of stress and strain on the magnet. Purposefully causing the cables to shift, even if into a more stable position, leaves the magnet less mechanically robust. Therefore, it is also desirable to give feedback to the manufacturers of the magnets so that they can rework the design or production process and produce dipoles that require less training. Successful response to proper feedback would result in fewer magnets being rejected, which would, of course, lead to money and time being saved.

The feedback that would be most valuable to the manufacturers is an analysis of the locations where the quenches originate in each magnet. Patterns in the positions of the quench origins could point to weak points in the cables' placement and give the manufacturers a place to begin re-engineering. Since the quench process is not visible to the human eye and does not leave visible traces of its place of origin, some indirect means must be used to make this analysis. That is where the LQA, or Local Quench Antenna, comes into play. The LQA is basically a set of coils of wire that provide a voltage signal as a measure of the change in the magnetic field (i.e., the flux in the coils) due to the spreading of the normal-conducting zone. This voltage signal has been dubbed the "quench signal." Analyzing this signal can help in understanding the quench, including its propagation and location.

To turn the raw data of the quench signals into information about quench propagation and quench location, a model must be developed to explain, mathematically, the connection between a quench and its LQA signal. By accurately simulating quench signals, one may be confident in relating actual signals to corresponding modelized scenarios. A model has been developed, simulated data has been gathered, and the pursuant analysis of quench propagation and location has been made. What follows is a description of the motivation of the model (first in the form of objectives and then in the form of physical motivation), a description of the actual model and its simulated data, and the results and conclusions drawn from the simulations.

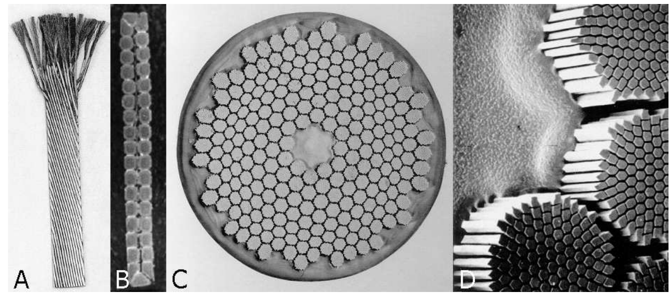

Fig. 1. **A.** A superconducting Rutherford cable; **B.** A cross-section of the cable (looking upward from the bottom of picture A), showing 36 strands; **C.** A cross-section of an individual strand of the cable, with the arranged NbTi filaments in a copper matrix; **D.** A close-up of the NbTi filaments

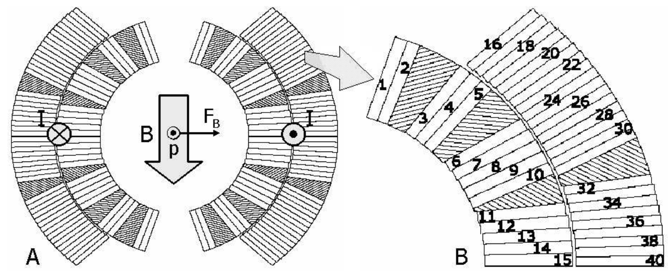

Fig. 2. **A.** Each rectangle is a cross-section of a cable, like the cross-section in Figure 1, picture B. The special arrangement of the current, I, creates the downward dipole magnetic field, B, which forces (F_B) the proton beam, p, to turn along the accelerator's path. **B.** This cable numbering system is mirrored for each of the other quadrants.

B. Objectives

First and foremost, the goal of this exercise in modeling is to be able to take the raw data of the quench signals and extract information about the location of the origin of the quench and characteristics of its propagation. This goal is directly related to providing feedback to the companies who are producing the dipoles, as was stated in the Background. Secondary goals of this project include developing tools and techniques for the discipline of building, testing, and maintaining superconducting dipole magnets. Gaining a better knowledge of the quench phenomenon will also be beneficial to any future technologies utilizing superconducting cables in the same fashion as the LHC dipoles.

II. Physical Situation

Before describing the quench model, the physical details of the LHC dipoles, a quench, and the LQA (Local Quench Antenna) should be examined so that the model is justified.

A. LHC Dipoles

The LHC dipoles' magnetic fields are created using superconducting Rutherford cables. The cables are made of strands of wire that are twisted around each other, each wire being an arrangement of NbTi filaments in a copper matrix (Fig. 1). When the cable is superconducting, it is the NbTi filaments that carry the current. When normal-conducting, the NbTi in the cable is more resistive than the copper and so the copper carries the current instead of the NbTi. The cables form two concentric layers around each beampipe in the dipole (Fig. 2). Since the two beampipes carry proton beams moving in opposite directions, the dipole fields in each pipe must be pointing in the opposite direction to keep both beams in the same circular (27-km) path. Thus, the current directions in the cables for one beampipe are opposite those of the cables for the other pipe (Fig. 3). The two layers of cables are made using two slightly different types of cable: the inner layer consists of cables composed of 28 strands while the outer layer consists of smaller cables composed of 36 strands. Actually, each layer is made of only one cable, with the cable looped around its beampipe in the manner illustrated (Fig. 3). (Being made of filamented wires allows the cables to be bent without being damaged.) Each loop of a cable is given a number (Fig. 2) to distinguish the parts of the cable, and each loop is specified as either being above or below the beampipe. For naming purposes, once a side of the beampipe is specified, each loop is referred to as a cable, e.g. "cable 1." Further specifications are whether the cable is on the "external side" or the "internal side" of the accelerator's circular path or whether its beampipe is the "upstream" or "downstream" pipe. (Further complicating the situation, the beampipes switch roles of being "upstream" or "downstream" along the accelerator.)

As designed, the special configuration of the cables creates a

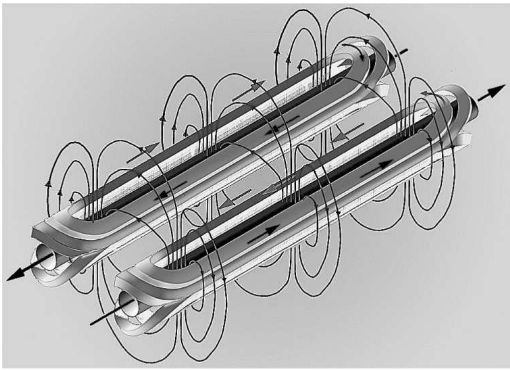

Fig. 3. The superconducting cables loop around the beampipes inside the dipole in the manner shown. For each beampipe the upper half of the inner layers of cables is drawn blocked together, as is the upper half of the outer layers and the lower halves of the inner and outer layers. The left beampipe is on the external side of the dipole and the right beampipe is on the internal side.

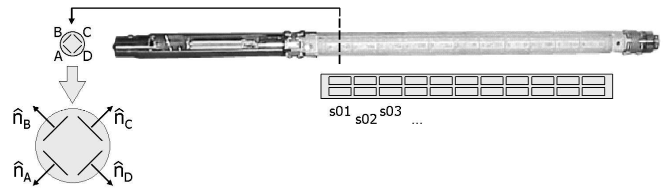

Fig. 4. The rod-shaped Local Quench Antenna (LQA), 36 mm in diameter, is placed inside a beampipe, 40 mm in diameter. Four LQAs are placed in the ends of the two beampipes in a dipole. (Larger antennae revealed that most quenches occur near the ends of the dipole, where the cables are bent, rather than the middle of the dipole.) The coil-sets are named from $s01$ to $s11$, where $s01$ is closest to the mouth of the beampipe. (In the lengthwise cross-section, coils B an C are overlapping, as are coils A and D.) The normal vectors that determine the sign of the flux in the coils are shown in the magnified cross-section. Given a particular coil-set, coils A and C provide a voltage signal V_{AC}, and coils B and D provide a voltage signal V_{BD}. So each of the eleven coil-sets has two asssociated quench signals.

dipole magnetic field that is homogeneous within a beampipe[1]. Of course, the field outside a beampipe is not homogeneous; it changes direction and becomes weaker farther away from the beampipe (Fig. 3). Thus, magnetoresistive effects within the cables are not homogeneous. This is a concern mainly when a cable is normal-conducting because the copper in the cable is more affected by magnetoresistance. The copper matrices are in contact with each other and so form something like a “swiss-cheesed” cable that can be considered nearly continuous. Thus if magnetoresistivity increases transversally across the cable, then the current density in the copper decreases across the cable. The NbTi, on the other hand, is a collection of separate filaments that wrap around each other. Over distances larger than the length required for one strand to wrap once around the cable, the filaments have the same inductive characteristics and are indistinguishable from each other. So, assuming that the whole cable is superconducting, the current distribution in that cable should be homogeneous even if the magnetic field is not.

B. Quench

The quenches of interest for this project are the ones that are initiated by friction between the cables. The cables are carrying currents of about 12 kA, and as the cables are situated in the high magnetic fields produced by these currents, the cables feel large forces. Of course, these forces vary throughout space as the magnitude of the magnetic field varies, so some cables feel much more force than others. It turns out that the cables of particular concern are the first few cables at the edges of each layer (cables 1-5 and 16-20), where the field is strongest and forces are on the order of 1 kN/cm for each cable.

Once some piece of the cables rub together, heat is absorbed. This may put the temperature of that piece above the critical temperature T_C, where T_C is a function of the current density and the magnetic field at that location. Then that piece of the cable is normal-conducting and resistive; therefore, it begins to dissipate heat and cause the area around itself to become normal-conducting. This irreversible process continues within the cable, spreading the normal-conducting zone in both directions along the cable. The two boundaries between the normal-conducting zone and superconducting zone, or the quench fronts, quickly reach an essentially constant velocity v_q as they move along the quenching cable. This quench velocity, as it is called, can be anywhere from 10 m/s to 30 m/s.

Since the cables are supplied by a current source, there remains a 12 kA current throughout the cable as more and more of the cable becomes normal-conducting. This would lead to thermal damage of the dipole if measures were not taken to halt the process. However, halting the quench requires shutting down the dipole, which means the large amount of energy in the form of the magnetic field must somehow be allowed to quickly dissipate as the current is ramped down. “Quench heaters” help to accomplish this by pre-empting the quenching process and bringing the whole cable into the normal-conducting phase, thus dissipating the energy evenly over the whole dipole. The quench heaters are activated as soon as voltmeters connected to the cables surpass a critical value that indicates the cables have acquired too much resistance.

C. Antenna

The LQA is essentially a collection of 44 small coils of wire. Each coil is in the shape of a 4-cm by 1-cm rectangle and has 400 turns, making it 1 mm thick. The coils are arranged in sets of four, with the four coils place at 90 degree angles from each other and 45 degrees from the horizontal. The positioning and naming convention of the coils is illustrated (Fig. 4).

The coils are wired so that if the magnetic field is increasing in the direction away from the center of the LQA, a positive voltage is induced. In addition, the coils opposite each other are connected in series, adding their voltages, so as to maximize the signal-to-noise ratio. That is, if the overall dipole field

[1]Since the dipole magnets have two beampipes, each with their own magnetic dipole fields, perhaps a better name would be “double-dipole magnets.”

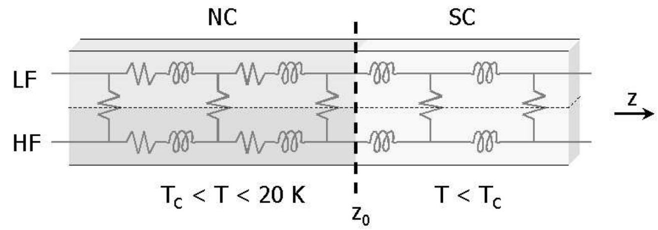

Fig. 5. This is the model, with the horizontal direction, or z-axis, greatly magnified. The normal-conducting (NC) zone is on the left and the superconducting (SC) zone is on the right. For this particular cable, the non-homogeneous magnetic field results in a high-field (HF) region below a low-field (LF) region. Each region takes up half of the cable, and the wires are centered in each of these halves. The quench front is located at the position z_0 along the z-axis, which runs parallel with the cables, and $z_0(t) = v_q t$.

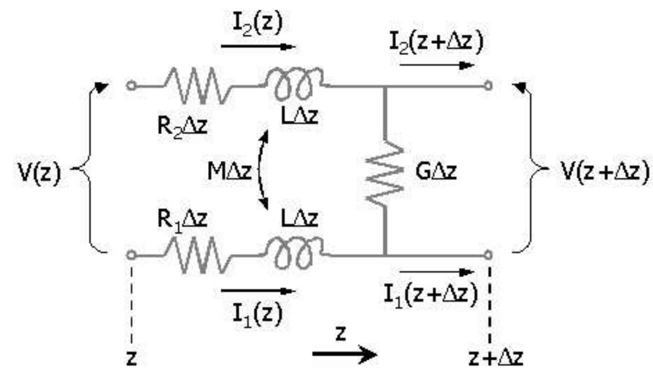

Fig. 6. A schematic of an infinitessimal length Δz of the two wires.

is changing homogeneously, then the two induced signals will cancel out. Eleven of these sets are lined up lengthwise inside the LQA, spaced at 4-cm intervals. (The coils are actually slightly less than 4 cm long, short enough to allow half of a millimeter of space between the sets.) The part of the LQA encasing the coils is made of fiberglass (G11), and the metalic part that holds the mechanical and electrical connections is made of titanium. Titanium has low magnetic permeability, so it does not experience large forces while near the beampipe.

III. Model

Now, with a good grasp of the physical set-up, the model should be easily understood, and the simplifications made therein to extract the essential physics should seem reasonable.

A. Quenching Cable

Since the quenches are detected with coils that measure changes in the magnetic field, and since the current in the cables is the source of the magneic field, the model must reproduce the changing current distribution associated with a quench. However, since the current distribution only changes locally around and within the normal-conducting zone, and since the normal zone does not have much time to expand before the quench heaters are activated, only one loop of the quenching cable has to be examined. To simplify the problem and to attack it with a divide-and-conquer strategy, the model describes the propagation of only one quench front, where its velocity has already reached a constant value and the other front is too far away to be detected. Also, the model quench front only traverses the straight section of the cable, with the turns at the ends of the loop assumed to be far away.

To capture the essential physics of this problem and to simplify it further, the quenching Rutherford cable is represented as two straight wires with discrete resistivities. The wires also share conductive and inductive properties with each other. This model is represented pictorially in Figure 5.

The temperature is assumed to be between T_C and 20 K in the normal-conducting zone. Since resistivity changes very little in this range of temperatures, R_1 and R_2 are assumed to be constant, given a particular cable with particular magnetic fields. (The values for R_1 and R_2 are calulated as what they would be at 10 K.) In the superconducting zone, R_1 and R_2 are zero, so there are no longitudinal resistors drawn. The conductivity between the wires is G, their self-inductivity is L, and their mutual-inductivity is M. After defining several variables with a schematic (Fig. 6), a few relationships become aparent:

$$I_1(z,t) + I_2(z,t) = I_{tot}, \tag{1}$$

the current I_1 in wire 1 and the current I_2 in wire 2 remains constant since their is no collection, production, or destruction of charge along the wires;

$$\begin{aligned}\Phi_{node} &= I_1(z{+}\Delta z,t) - I_1(z,t) - G\,\Delta z\, V(z{+}\Delta z,t) \\ &= 0,\end{aligned} \tag{2}$$

the current density flux Φ_{node} at a node such as the lower node in Figure 6 is zero because there is no collection, production, or destruction of charge at a node; and

$$\begin{aligned}V_{loop} &= V(z{+}\Delta z,t) \\ &\quad + \Big[L\,\Delta z\,\dot{I}_2(z,t) + M\Delta z\,\dot{I}_1(z,t)\Big] \\ &\quad + R_2\,\Delta z\, I_2(z,t) - V(z,t) - R_1\,\Delta z\, I_1(z,t) \\ &\quad - \Big[L\,\Delta z\,\dot{I}_1(z,t) + M\Delta z\,\dot{I}_2(z,t)\Big] \\ &= 0,\end{aligned} \tag{3}$$

the change in electric potential V_{loop} around a circuit loop is zero because of the conservation of energy in the circuit.

Taking the limit as Δz goes to zero, Equation 2 becomes ${I_1}' = G\,V$ and Equation 3 becomes $V' = R_1\,I_1 - R_2\,I_2 + (L{-}M)\,\dot{I}_1 - (L{-}M)\,\dot{I}_2$. (The prime in ${I_1}'$ refers to the derivative with respect to z, and the dot in $\dot{I}_1$ refers to the derivative with respect to t.) Further, if a difference current, or "current redistribution," $i(z,t)$ is defined,

$$i(z,t) \equiv \frac{1}{2} I_1(z,t) - \frac{1}{2} I_2(z,t), \tag{4}$$

so that

$$I_1(z,t) = \frac{1}{2} I_{tot} + i(z,t) \tag{5}$$

and

$$I_2(z,t) = \frac{1}{2} I_{tot} - i(z,t), \tag{6}$$

then the two differential equations consolidate to

$$i'' - 2(L-M)\,G\,\dot{i} - (R_1+R_2)\,G\,i = (R_1-R_2)\,G\,I_{tot}/2, \tag{7}$$

thus eliminating V, I_1, and I_2 from the equation.

Since the quench velocity is constant and the signals induced in each coil-set look the same as the quench front moves past the coil-sets, it must be that $i(z,t)$ (and all of the other functions of z and t mentioned so far) is a traveling waveform; that is, $i(z,t) = i(z - v_q t)$. In that case, it is true that

$$\dot{i} = -v_q\, i'. \tag{8}$$

Using this information and simplifying the inductive part of Equation 7 to an equivalent inductivity L_{eq} yields

$$i'' + L_{eq}\,G\,v_q\,i' - (R_1+R_2)\,G\,i = (R_1-R_2)\,G\,I_{tot}/2. \tag{9}$$

Applying the boundary conditions that i should remain finite and that its derivative should be continuous at the quench front, the solution ends up being

$$\begin{aligned} i(z) \;&=\; \frac{\Delta I}{2}\left[\mathcal{U}(-z)\left(1 - \frac{\lambda_n}{\lambda_n+\lambda_s} e^{z/\lambda_n}\right)\right. \\ &\qquad \left. + \,\mathcal{U}(z)\frac{\lambda_s}{\lambda_n+\lambda_s} e^{-z/\lambda_s}\right], \end{aligned} \tag{10}$$

where z is replacing the argument $z - v_q t$ (or $t = 0$), $\mathcal{U}$ is the Heaviside unit step function,

$$\Delta I = \frac{(R_1-R_2)}{(R_1+R_2)} I_{tot}, \tag{11}$$

$$\frac{1}{\lambda_s} = L_{eq}\,G\,v_q, \tag{12}$$

and

$$\frac{1}{\lambda_n} = \frac{1}{2}\left(\sqrt{\left(\frac{1}{\lambda_s}\right)^2 + 4(R_1+R_2)G} - \frac{1}{\lambda_s}\right). \tag{13}$$

Given that the values of the parameters R_1, R_2, and G depend upon the magnitudes of the magnetic field, B_1 and B_2, at the positions of the two wires, the current redistribution $i(z - v_q t)$ is different for each cable. The value of L_{eq}, on the other hand, is a matter of geometry and is calculated to be on the order of 10^{-7} H for all cables. For one of the likely quenching cables, cable 1, the values of R_1 and R_2 are respectively taken to be 65.2 $\mu\Omega$/m and 60.5 $\mu\Omega$/m, and the conductivity G is taken to be 2.25×10^7 S/m. Thus, assuming that v_q = 30 m/s and I_{tot} = 11.85 kA, the current distributions in the two wires take on the curves shown in Figure 7. To illustrate how the redistribution is affected by a change in the parameters, some additional curves are shown in Figures 8 and 9.

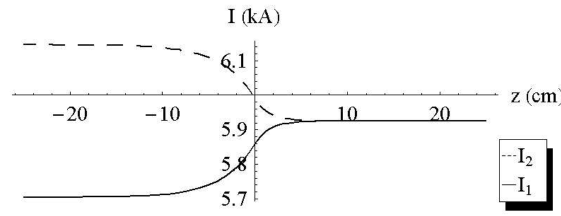

Fig. 7. Current distribution in the two wires at $t = 0$.

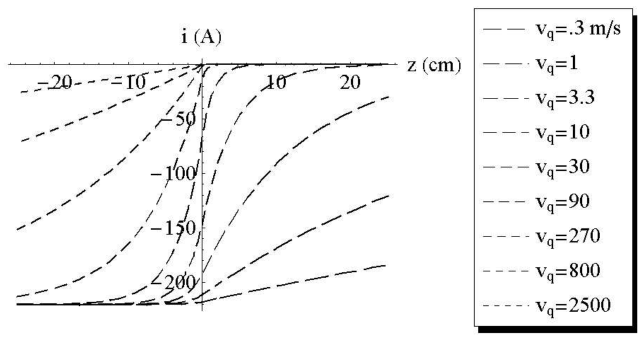

Fig. 8. For cable 1, keeping G at the same value (2.25×10^7 S/m) and changing v_q produces these current redistributions.

B. Antenna

It is assumed that the coils in the LQA are essentially two-dimensional, i.e., that they are $l \times w$ rectangular loops with no thickness, where l = 4 cm and w = 1 cm. The coils of wire are looped N_t times, where $N_t = 400$. It is also assumed that their 1-cm widths are small enough that the magnetic field can be taken as constant along that dimension. The values for the magnetic field vector throughout a given coil are thus taken along a line down the center of the coil.

Mathematically, the quench signal $V(t)$ for a single coil due to the changing current distribution in wire 1 alone is expressed below:

$$V(t) = -\dot{\Phi}_B(t), \tag{14}$$

the negative time derivative of the magnetic flux Φ_B in the coil

$$= -\partial_t\left[N_t \int_S \mathbf{B}(z', t)\cdot d\mathbf{A}\right], \tag{15}$$

where the magnetic field B is a function of z' taken along the

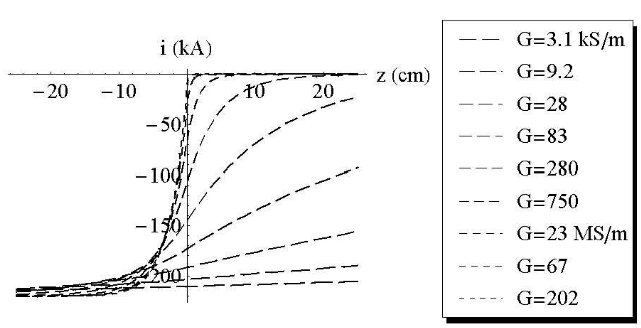

Fig. 9. For cable 1, keeping v_q at the same value (30 m/s) and changing G produces these current redistributions.

center of the coil and is integrated over the surface S of the coil, with $d\mathbf{A}$ pointing in the assigned normal direction $\hat{\mathbf{n}}$ of the coil

$$= -\partial_t \left[N_t \int_{-l/2}^{l/2} \mathbf{B}(z',t) \cdot \hat{\mathbf{n}}\, w\, dz' \right], \tag{16}$$

because the surface integral simplifies to a line integral

$$= -\partial_t \left[N_t \int_{-l/2}^{l/2} \left(\int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} \frac{I_1(z - v_q t)\, d\mathbf{z} \times \mathbf{r}}{r^3} \right) \cdot \hat{\mathbf{n}}\, w\, dz' \right], \tag{17}$$

where the magnetic field B is calculated using Biot-Savart's Law, with z taken as a position along wire 1 and with $\mathbf{r}$, a function of z and z', taken as the vector extending from the position along the wire at z to the position along the center of the coil at z'

$$= -\frac{\mu_0}{4\pi} \partial_t \left[N_t\, w\, (\hat{\mathbf{z}} \times \mathbf{r}) \cdot \hat{\mathbf{n}} \int_{-l/2}^{l/2} \left(\int_{-\infty}^{\infty} I_1(z - v_q t) \frac{1}{r^3} dz \right) dz' \right],$$

after pulling out the constants from the integrals (and since $\mathbf{r}$ *per se* should not be taken out of the integral, let $(\hat{\mathbf{z}} \times \mathbf{r})$ be replaced by its equivalent, $(\hat{\mathbf{z}} \times \mathbf{d})$, where $\mathbf{d}$ is the distance vector, perpendicular to the coil's axis and wire 1, that gives the distance d between the coil and wire 1)

$$= -\frac{\mu_0}{4\pi} \partial_t \left[N_t\, w\, (\hat{\mathbf{z}} \times \mathbf{d}) \cdot \hat{\mathbf{n}} \int_{-\infty}^{\infty} \left(\int_{-l/2}^{l/2} I_1(z - v_q t) \frac{1}{r^3} dz' \right) dz \right],$$

after switching the order of integration

$$= -\frac{\mu_0}{4\pi} \partial_t \left[N_t\, w\, (\hat{\mathbf{z}} \times \mathbf{d}) \cdot \hat{\mathbf{n}} \int_{-\infty}^{\infty} I_1(z - v_q t) \left(\int_{-l/2}^{l/2} \frac{1}{r^3} dz' \right) dz \right],$$

after pulling out I_1, which is constant with respect to z'

$$= -\frac{\mu_0}{4\pi} \partial_t \left[\int_{-\infty}^{\infty} I_1(z - v_q t) \left(\int_{-l/2}^{l/2} \frac{N_t\, w\, (\hat{\mathbf{z}} \times \mathbf{d}) \cdot \hat{\mathbf{n}}}{r^3} dz' \right) dz \right],$$

where all the geometry-dependent[2] constants are grouped with the geometry-dependent integral

$$= -\frac{\mu_0}{4\pi} \partial_t \left[\int_{-\infty}^{\infty} I_1(z - v_q t)\, h(z)\, dz \right], \tag{18}$$

calling the geometry-dependent integral h and the "geometric coupling function," or just the "coupling function"

$$= -\frac{\mu_0}{4\pi} \partial_t (I_1 * h)(-v_q t), \tag{19}$$

the convolution of I_1 and h, by definition

$$= -\frac{\mu_0}{4\pi} \left(\dot{I}_1 * h \right)(-v_q t), \tag{20}$$

since only I_1 is a function of time

$$= -\frac{\mu_0}{4\pi} \left(\dot{i} * h \right)(-v_q t), \tag{21}$$

because of the relation in Equation 5

$$= \frac{\mu_0}{4\pi} v_q \left(i' * h \right)(-v_q t), \tag{22}$$

because of the relation in Equation 8.

So the quench signal is simply a convolution of the derivative of the current redistribution in the quenching cable with the pertinent geometric coupling function. The geometric coupling function h is explicitly calculated below (letting $N_t\, w\, (\hat{\mathbf{z}} \times \mathbf{d}) \cdot \hat{\mathbf{n}} = g$):

$$h(z) = \int_{-l/2}^{l/2} \frac{g}{r^3} dz' \tag{23}$$

$$= \int_{-l/2}^{l/2} \frac{g\, dz'}{(d^2 + (z - z')^2)^{3/2}} \tag{24}$$

$$= g \left(\frac{l/2 - z}{d^2 \sqrt{(l/2 - z)^2 + d^2}} + \frac{l/2 + z}{d^2 \sqrt{(l/2 + z)^2 + d^2}} \right). \tag{25}$$

Since pairs of coils are connected together in series, the resulting signal is a sum of the signals from each coil due to each wire. For example, coils A and C are in series, so the resulting signal V_{AC} is

$$V_{AC}(t) = V_{A1}(t) + V_{A2}(t) + V_{C1}(t) + V_{C2}(t) \tag{26}$$

$$= \frac{\mu_0}{4\pi} v_q \left[\left(i' * h_{A1} \right)(-v_q t) + \left(i' * h_{A2} \right)(-v_q t) + \left(i' * h_{C1} \right)(-v_q t) + \left(i' * h_{C2} \right)(-v_q t) \right] \tag{27}$$

$$= \frac{\mu_0}{4\pi} v_q \left(i' * [h_{A1} + h_{A2} + h_{C1} + h_{C2}] \right)(-v_q t) \tag{28}$$

$$= \frac{\mu_0}{4\pi} v_q \left(i' * H_{AC} \right)(-v_q t), \tag{29}$$

where H_{AC} is the total geometric coupling function for coils A and C with respect to a particular cable. The quench signal V_{BD} and its coupling function H_{BD} come about in the same manner.

C. Simulated Signal

Using the equations derived from the model, a program written in $Mathematica$ has generated the quench signals V_{AC} and V_{BD} for all of the 40 cables in one quadrant of the array of cables around one beampipe. Because of the symmetry of the setup, one quadrant of one beampipe describes all quadrants for both beampipes. For one cable (cable 1) the coupling function, the derivative of the current redistribution, and the resulting simulated signal are shown (Fig. 10, pictures A, B, and C).

Examining the equations and the graphs, the more the current redistribution resembles the Dirac delta, the more the quench signal approximates the coupling function. And, vice versa, the more the coupling function resembles the Dirac delta, the more the quench signal approximates the current redistribution. So a sharper coupling function is more desirable for capturing the essence of the quench probagation. Smaller coils in the LQA will yeild sharper coupling functions, but the coils should not be so small that its signals have small magnitudes on par with the noise.

IV. Results

The results of the modeling and simulation are conceptual, procedural, and fullfilling of the objectives set forth earlier.

A. Deconvolution

One useful result of this model and the success of its simulations is the realization that a simple convolution can relate the quench signal to the changing current distribution via a geometical coupling function. So long as the pattern of the current distribution (or the "redistribution") travels as a waveform with a constant velocity, and so long as the quench front is propagating along the straight section of the quenching cable, a convolution will describe the interaction of the quench with the LQA. This idea works even for more complicated models of the quenching cable that have more than two wires.

Once the convolving relationship is discovered, though, it is not long before the idea of deconvolving a measured quench signal arises. With a good knowledge of the coupling

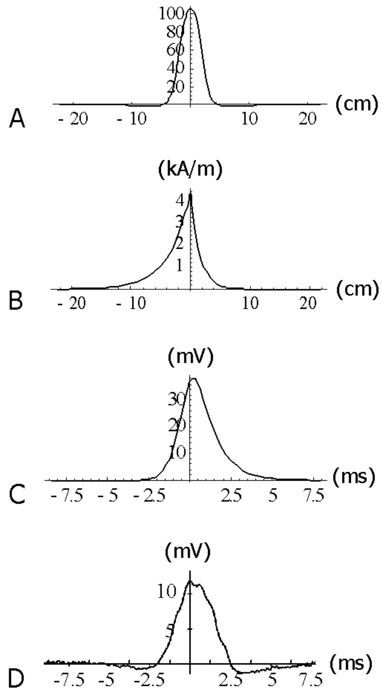

Fig. 10. **A.** This is the geometric coupling function H_{AC} for cable 1. The assumptions for these graphs were that the quench occurred in cables surrounding a beampipe such as the left beampipe in Figure 3, with the quench on the internal side of the top loops and the quench front of interest traveling in the same direction as the current. Note that H_{AC} is unitless. **B.** This is i', the derivative of the current redistribution for cable 1. **C.** This is the resulting quench signal V_{AC}, the convolution of the graphs in the previous two figures. **D.** This is an actual quench signal measured by the LQA, or, more precisely, two coils of the LQA. The magnitude of the simulated signal is off by a factor of approximately three; this may be due to a slight mischaracterization of the magnetoresistiviy of the cable.

function, a signal can be translated by deconvolution into the "actual" current distribution, according to the particular model that is used in deriving the coupling function. Of course, once a model is chosen, the coupling function *is* well known because the LQA is well known and controllable. Then, after deconvolution, the "actual" current distribution can be compared with the modelized current distribution to determine the quality of the model. So a result of this model has been to find a second way of analyzing the raw data of the quench signals.

B. Quench Characterization

One way to determine how well the model characterizes the quench process is to compare the simulated quench signals with the actual signals. (Another way, as just described in the preceding section, is to compare the deconvoluted actual signals with the modelized current redistributions.) So, an actual signal is included in Figure 10 (picture D), which can be compared to the simulated signal in the same figure (picture C).

The peak of the simulated signal is off from this peak by a factor of three. This is probably due to a slight mischaracterization of the resistive and magnetoresistive properties of the cables. The simulated signal also does not include the dip below zero volts on the right side of the peak. This dip is more prominent in other signals and is due to thermal activity in the cable: the less-resistive part of the cable (or "wire") carrying more current heats up more quickly than the more-resistive part, and becomes more resistive so that the resistivities eventually equalize and the current redistributes itself evenly over the cable again. A more complicated model and program (known as SPQR), which includes thermal characteristics and equations, does account for this dip. The dip, however, is not immediately of concern; the present model describes the most prominent aspects of the quench signal.

C. Quench Locationing

Two questions that arise after quench measurements are taken are "which cable was the one that quenched?" and "where in that cable did the quench originate?" As for the question of which cable is the quenching cable, it can be broken into stages, such as which loops quenched (the loops above or below the beampipe?) and on which side did the quench occur (the internal or external side?). Since the quench heaters are activated by a voltage signal that is associated with either the top loops or the bottom loops, it is known whether the quench occurred in the top or bottom half of the cables. The results of the simulations give an answer as to which side the quench was on and helps in determining which cables might have quenched.

The strength of a quench signal is partly determined by the position of the quenching cable, specifically, the distance of the quenching cable from the LQA coils of interest and the angles it makes with the coils. So comparing the peak voltages of the two signals for a particular coil-set could possibly tell something about whether the quench occurs on one side or another. The simulations have revealed that a ratio of the peak voltages does indeed indicate which side has quenched. Depending on whether the absolute value of V_{AC}/V_{BD} is greater than or less than one, the quench is on one side or the other. This rule is true for all cables except one (cable 21), which just happens to have the right positioning to be different from the rest. These ratios can be seen in Table I. The data in the table were calculated assuming the quench occurred in cables surrounding a beampipe such as the left beampipe in Figure 3, with the quench on the internal side of the top loops and the quench front of interest traveling in the same direction as the current. The sign of the ratio is always negative, but the magnitude of the ratio gives some indication as to which cable was the quenching cable, especially if the choice of cables is limited to those of highest concern (cables 1-5 and 16-20).

Now, as for the question of where the quench originates in the quenching coil, an answer can be found if an additional fact outside of the model is taken into consideration. When a quench is caused by friction between cables due to the sudden shift of the cables, the shift in the position of the cables themselves provide a change in the spacial current distribution, thereby inducing a signal in the LQA. This signal appears as

Number of Quenching Cable	$V_{AC,peak}/V_{BD,peak}$
1	-2.66
2	-3.30
3	-4.41
4	-4.84
5	-5.14
6	-12.62
7	-6.87
8	-5.40
9	-4.46
10	-3.68
11	-3.03
12	-2.36
13	-1.85
14	-1.46
15	-1.15
16	-2.00
17	-2.00
18	-1.97
19	-1.89
20	-1.66
21	-0.72
22	-3.95
23	-2.78
24	-2.52
25	-2.38
26	-2.26
27	-2.17
28	-2.08
29	-1.99
30	-1.91
31	-1.82
32	-1.73
33	-1.63
34	-1.53
35	-1.44
36	-1.35
37	-1.26
38	-1.18
39	-1.11
40	-1.04

TABLE I

VOLTAGE-PEAK RATIOS FOR A GIVEN SIDE

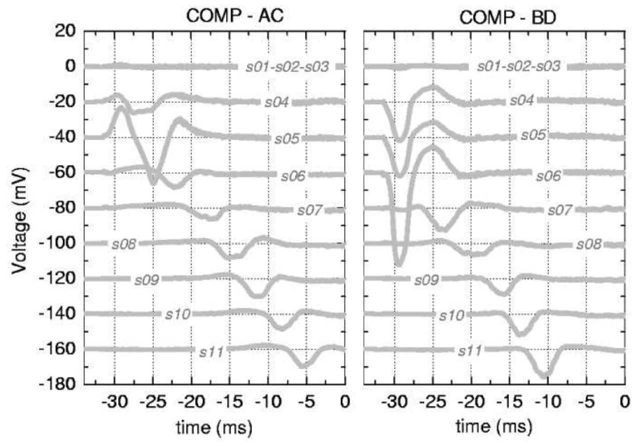

Fig. 11. The raw data here is filtered, so a simultaneous voltage spike is not seen in the coils. COMP-AC is another name for V_{AC}, the compensated signals from coils A and C, as COMP-BD is another name for V_{BD}. The first points to notice about the data are that the two sets of signals both dip downward and the dips for given coil-set are about 5 ms earlier in V_{BD} than in V_{AC}. This can be explained by the propagation of two quench fronts, one of which travels around the bend in the cables to the other side of the beampipe. So there are really two quench signals in V_{AC} and two signals in V_{BD}, where the signal from a given quench front is simultaneously present in both V_{AC} and V_{BD}, but the positive signals are much smaller than the negative, dipping ones.

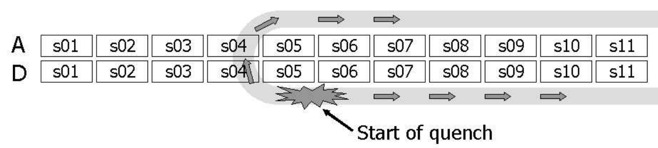

Fig. 12. This is the interpretation of the data in Figure 11, where the quench orginates near s05 and s06 on one side of the loop of cable and one of the quench fronts travels to the other side. The quench is known to have occurred in the bottom cables, so the closest coils, coils A and D, are drawn.

a voltage spike in the LQA that is essentially simultaneously detected in all coil-sets. Since it is simultaneous, it provides the time at which the quench occurs. Then, once the quench front propagates along the cable and past the LQA, the direction of quench propagation can be detected and the velocity of the quench, v_q, can be measured. Thus, the approximate location of the origin of the quench can be traced backwards using v_q and the amount of time since the beginning of the quench. On the other hand, if the quench happens to initiate in the region that the LQA occupies, then its starting point can be directly deduced (e.g., between s05 and s06 in Fig. 11).

V. CONCLUSION

The proposed goals of characterizing quenches and their signals and locating the origin of quenches have successfully been met with the two-wire model and its simulations. Of course, improvements on this model, such as increasing the number of discrete wires or including dynamic thermal activity (as has been done with the program SPQR), will improve the results, but this model captures the essential physical phenomena that produce the quench signals. The model also provides insight into the analysis of actual signals using the idea of convolution and deconvolution for steady-state quench propagation that has reached a constant velocity. It is a step closer to efficient production of LHC dipoles and mastering the technology of superconducting electromagnets.

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