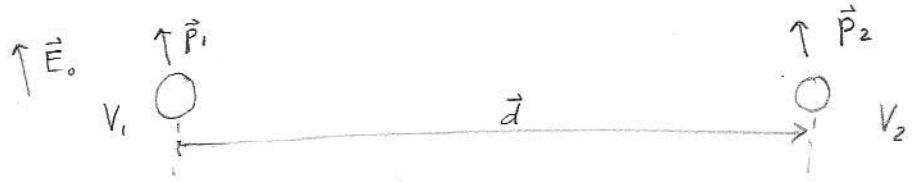
9. *Electricity and Magnetism* (Fall 2005)

Two small pieces of uncharged, continuous, polarizable matter (for example, glass) are placed in a region in which there is an externally generated, uniform field E_0. The two small pieces of matter have volumes V_1 and V_2 and electrical susceptibilities χ_1 and χ_2, respectively. If they are separated by a distance d, such that $d^3 \gg V_1$ and $d^3 \gg V_2$, find the energy associated with the interaction between the two pieces (that is, the part of the energy that depends on d.)



$$\vec{p}_1 = V_1 \vec{P}_1 = V_1 \varepsilon_0 \chi_1 \vec{E}_0 \qquad \vec{p}_2 = V_2 \vec{P}_2 = V_2 \varepsilon_0 \chi_2 \vec{E}_0$$

$$W_{12} = K_e \frac{\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{d})(\vec{p}_2 \cdot \hat{d})}{d^3} = K_e \frac{p_1 p_2 - 3 p_1 p_2 (\hat{E}_0 \cdot \hat{d})^2}{d^3}$$

$$= K_e \frac{p_1 p_2}{d^3}\left[1 - 3(\hat{E}_0 \cdot \hat{d})^2\right] = \frac{1}{4\pi\varepsilon_0} \frac{\varepsilon_0^2 E_0^2 V_1 V_2 \chi_1 \chi_2}{d^3}\left[1 - 3(\hat{E}_0 \cdot \hat{d})^2\right]$$

$$= \frac{\varepsilon_0 E_0^2 V_1 V_2 \chi_1 \chi_2}{4\pi d^3}\left[1 - 3(\hat{E}_0 \cdot \hat{d})^2\right]$$