12. *Electricity and Magnetism* (Fall 2004)

(a) Show that the annihilation of an electron and a positron can produce a single massive particle (say, X), but not a single photon.

(b) A positron beam of energy E can be made to annihilate against electrons by hitting electrons at rest in a fixed-target machine or by hitting electrons moving in the opposite direction with the same energy E in an electron-positron collider (colliding-beam accelerator). Show that the minimum energy E_{min} of a positron beam needed to produce neutral particles X of mass $M \gg m_e$ (where m_e is the electron rest mass) is much greater in a fixed-target machine than in a collider.

a) $\xrightarrow{e^-} \xleftarrow{e^+}$

$\gamma \cdot$ X

X. ✓

In the center-of-mass frame the total momentum is zero, so the resultant particle must have zero momentum. Any massless particles, including photons, always have nonzero instantaneous momentum, so a single photon cannot be the product. (The total momentum of multiple massless particles could add to zero, though.) A massive particle such as X can have zero momentum and so can be a product of this annihilation.

b)

(Lab frame)	Before	After	
Collider	$\xrightarrow[E_c]{e^+}$ $\xleftarrow[E_c]{e^-}$	X $\cdot$	E_c is the minimum energy to produce one X particle (so it is not excited)
Fixed	$\xrightarrow[E_f \ \vec{p}]{e^+}$ $\overset{e^-}{\cdot}$	$\xrightarrow[\vec{p}]{X}$	E_f is also the minimum (X not excited)

$$E_c + E_c = Mc^2 \quad \Rightarrow \quad E_c = \tfrac{1}{2}Mc^2$$

$$E_f + mc^2 = \sqrt{p^2c^2 + M^2c^4}$$

$$E_f^2 = p^2c^2 + m^2c^4 \quad \Rightarrow \quad p^2c^2 = E_f^2 - m^2c^4$$

$$(E_f + mc^2)^2 = p^2c^2 + M^2c^4 = (E_f^2 - m^2c^4) + M^2c^4$$

$$\Rightarrow \quad \cancel{E_f^2} + 2E_f mc^2 + m^2c^4 = \cancel{E_f^2} - m^2c^4 + M^2c^4$$

$$\Rightarrow \quad E_f = \frac{1}{2mc^2}\left[M^2c^4 - 2m^2c^4\right] = \frac{1}{2}\left(\frac{M^2}{m} - 2m\right)c^2 = \frac{1}{2}\left(\frac{M}{m} - 2\frac{m}{M}\right)Mc^2$$

$$\frac{E_f}{E_c} = \frac{M}{m} - 2\frac{m}{M} = \frac{M}{m}\left(1 - 2\left(\frac{m}{M}\right)^2\right) \approx \frac{M}{m} \gg 1 \quad \text{since } M \gg m \ \left(\text{and } \frac{m}{M} \ll 1\right)$$

$$\therefore \ E_f \gg E_c \ ✓$$