11. *Electricity and Magnetism* (Fall 2004)

Using general principles, find the radiated power in vacuum of a non-relativistic point charge q whose position is $\mathbf{r}(t)$. You do not need to find dimensionless proportionality constants (i.e., only find the dependence on q, $\mathbf{r}(t)$, and universal constants).

Let $a \equiv |\ddot{\vec{r}}|$

Assume the radiation electric field is proportional to a and to $\frac{1}{R}$ where R is the distance from the particle to the observation point:

$$E_a = A \frac{1}{4\pi\varepsilon_0} \frac{q}{R} a \quad \text{where } A \text{ is a constant of unknown dimensions}$$

$$[\vec{E}_a] = \left[A\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\right) R a\right] = [\vec{E}]\,[A R a] \Rightarrow [A] = \left[\frac{1}{Ra}\right] = \frac{s^2}{m^2} = \left[\frac{1}{c^2}\right]$$

$$\Rightarrow \quad E_a \propto \frac{1}{\varepsilon_0}\frac{q}{c^2}\frac{a}{R}$$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{2}\mathrm{Re}\left[\vec{S}_a \cdot R^2 \hat{n}\right] \qquad \vec{S}_a = \frac{1}{\mu_0}\vec{E}_a \times \vec{B}_a = \frac{1}{\mu_0 c}|\vec{E}_a|^2 \hat{k}$$

since $E_a = cB_a$ and $\hat{E}_a \times \hat{B}_a = \hat{k}$, the radiation propagation direction

Far away, $\hat{k} = \hat{n}$, and Ω is dimensionless

$$\Rightarrow \quad P \propto |\vec{S}_a| R^2 = \frac{1}{\mu_0 c} E_a^2 R^2 = \frac{1}{\mu_0 c}\frac{q^2 a^2}{\varepsilon_0^2 c^4 R^2} R^2$$

$$= \frac{c^2}{c}\frac{q^2 a^2}{\varepsilon_0 c^4} \qquad \text{since } \frac{1}{\mu_0 \varepsilon_0} = c^2$$

$$= \frac{1}{\varepsilon_0}\frac{q^2}{c^3} a^2$$

(The Larmor formula proportionality. ✓)