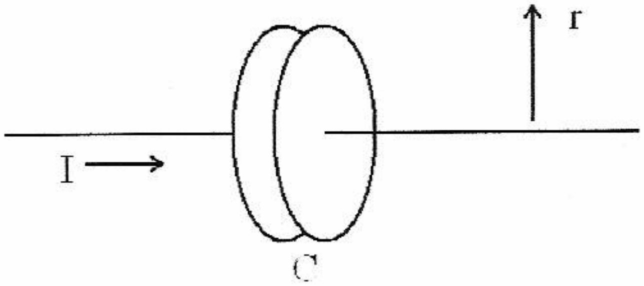
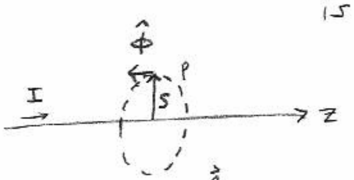
9. *Electricity and Magnetism* (Fall 2004)

A wire carrying current I is connected to a circular capacitor of capacitance C, as depicted in the figure. What is the magnetic field outside the wire, far from the capacitor (as a function of the distance r from the wire)? Using Maxwell's equations, explain why there is a magnetic field outside the capacitor. What is this magnetic field?



Far from wire: (the capacitor is negligible)



$$\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\partial_t \vec{E} = \mu_0 \vec{J} \quad (\vec{E} \to \vec{0})$$

$$\Rightarrow \oint_P \vec{B}\cdot d\vec{l} = \int_S \mu_0 \vec{J}\cdot d\vec{a} = \mu_0 I = B_\phi 2\pi s \quad \text{by cylindrical symmetry}$$

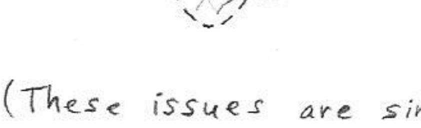
$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi} \quad \text{or} \quad \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

Field outside capacitor:

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\partial_t \vec{E}$$

$\Rightarrow$ A changing electric field can also be seen as a source for the magnetic field. There is a changing electric field in the capacitor.

To solve for the field outside the capacitor, one may solve for the changing electric field in the capacitor or note that the surface of integration S may be manipulated to avoid the fields in the capacitor:



$$\oint_P \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \frac{1}{c^2}\partial_t \int_S \vec{E}\cdot d\vec{a} = \mu_0 I \quad (\vec{E} \to \vec{0})$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

(These issues are simple when one assumes no fringing effects.)