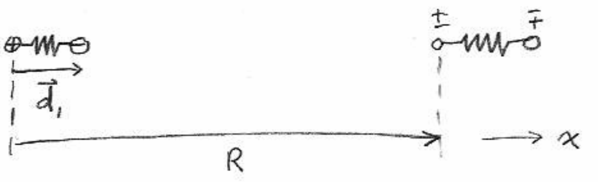
2. *Quantum Mechanics* (Fall 2004)

The van der Waals interaction between two neutral atoms is due to dipole-dipole interactions. Consider the following simplified 1-D model. Each atom consists of a fixed nucleus of charge $+e$ and electron of charge $-e$, bound by a harmonic spring. Two such oscillators are a distance R ($\gg$ size of the atom) apart. The Hamiltonian of the system consists of the two harmonic oscillator terms plus a dipole-dipole perturbation.

(a) Write the perturbation part of the Hamiltonian.

(b) Calculate the correction to the energy of the unperturbed ground state. This is the van der Waals interaction potential. (Hint: it should come out $\propto 1/R^6$.)

a) $H = H_1 + H_2 + H'$



$K = \frac{1}{4\pi\varepsilon_0}$

$H' = -\vec{p}_2 \cdot \vec{E}_1$

$$= -\vec{p}_2 \cdot \left[K \frac{3(\vec{p}_1 \cdot \hat{R})\hat{R} - \vec{p}_1}{R^3} \right] = -K \frac{3\, p_{1x} p_{2x} - p_{1x} p_{2x}}{R^3} = -2K \frac{d_{1x} d_{2x} e^2}{R^3}$$

$$= \mp 2K \frac{e^2}{R^3} d_1 d_2 \qquad \begin{array}{l} - \Rightarrow \text{aligned} \\ + \Rightarrow \text{antialigned} \end{array}$$

b) States $|n_1 n_2\rangle \quad n_1, n_2 \in \mathbb{N} = \{0, 1, 2, \dots\}$

$$d_i = d_0 (a_i^\dagger + a_i) \qquad d_0 = \sqrt{\frac{\hbar}{2m\omega}} \qquad H' = \mp 2K \frac{e^2}{R^3} d_0^2 (a_1^\dagger + a_1)(a_2^\dagger + a_2)$$

$\Delta E_{00}^{(1)} = \langle 00 | H' | 00 \rangle = 0$ since the $a^\dagger$'s and a's only connect states with $\Delta n_i = \pm 1$.

$$\Delta E_{00}^{(2)} = -\sum_{m \neq 0} \sum_{k \neq 0} \frac{|\langle mk | H' | 00 \rangle|^2}{E_{mk}^0 - E_{00}^0} = -\left(\frac{2Ke^2 d_0^2}{R^3} \right)^2 \sum_m \sum_k \frac{|\langle mk | (a_1^\dagger + a_1)(a_2^\dagger + a_2) | 00 \rangle|^2}{\hbar\omega (m + k - 1)}$$

$$= -\left(\frac{2Ke^2\hbar}{2m\omega R^3} \right)^2 \frac{1}{\hbar\omega} \left[\frac{|\langle 11 | 11 \rangle|^2}{(1+1-1)} \right] = -\frac{K^2 e^4 \hbar}{m^2 \omega^3 R^6}$$

(No need for degenerate perturbation theory in this case.)